

AD-A243 001



ARL-STRUC-TM-539

AR-006-621

2



DTIC
ELECTE
DEC 6 1991
S c D

DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Aircraft Structures Technical Memorandum 539

**DETERMINATION OF ANTISYMMETRIC END LOADS FROM BULK
STRESSES ON A RECTANGULAR PRISM**

by

J. BENNETT

Approved for public release

91-17083

© COMMONWEALTH OF AUSTRALIA 1991

SEPTEMBER 1991

91 12 4 079

This work is copyright. Apart from any fair dealing for the purpose of study, research, criticism or review, as permitted under the Copyright Act, no part may be reproduced by any process without written permission. Copyright is the responsibility of the Director Publishing and Marketing, AGPS. Enquiries should be directed to the Manager, AGPS Press, Australian Government Publishing Service, GPO Box 84, CANBERRA ACT 2601.

AR-006-621

**DEPARTMENT OF DEFENCE
DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION
AERONAUTICAL RESEARCH LABORATORY**

Aircraft Structures Technical Memorandum 539

**DETERMINATION OF ANTISYMMETRIC END LOADS FROM BULK
STRESSES ON A RECTANGULAR PRISM**

by

J. BENNETT

SUMMARY

A mathematical method is presented for the determination of the normal antisymmetric end loads from known values of the bulk stress on a lateral surface of a three dimensional rectangular prism. This extends previous work which considered only the symmetric loading case. A computer program has been developed to implement this method, and results for an arbitrarily chosen case are presented.



© COMMONWEALTH OF AUSTRALIA 1991

POSTAL ADDRESS:

**Director, Aeronautical Research Laboratory
506 Lorimer Street, Fishermens Bend 3207
Victoria Australia**

CONTENTS

CONTENTS SYMBOLS

1.	INTRODUCTION	1
2.	DIRECT SOLUTION	2
2.1	Loading Antisymmetric in Y	2
2.2	Loading Antisymmetric in Z	6
3.	INVERSE SOLUTION	10
3.1	Loading Antisymmetric in Y	10
3.2	Loading Antisymmetric in Z	12
4.	RESULTS AND DISCUSSION	14
4.1	Direct Results	14
4.2	Inverse Results	16
5.	CONCLUSION	16

REFERENCES
ACKNOWLEDGEMENTS
FIGURES
APPENDIX A
APPENDIX B
DISTRIBUTION LIST
DOCUMENT CONTROL DATA

Accession For	
NTIS GRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

SYMBOLS

x, y, z	Cartesian co-ordinate axes
$2a, 2b, 2c$	Length, depth, breadth of the prism
ν	Poisson's ratio
$\sigma_x, \sigma_y, \sigma_z$	Normal stress components in the x, y, z directions
$2k_1b, 2k_2c$	Lengths of loaded portion in the y and z directions
H_{mN}, K_{mN}, L_{lm}	Fourier coefficients (Loading Antisymmetric in y)
l, m, N	Indices of the Fourier series (Loading Antisymmetric in y)
H_{Mn}, K_{Mn}, L_{lM}	Fourier coefficients (Loading Antisymmetric in z)
l, M, n	Indices of the Fourier series (Loading Antisymmetric in z)
$f(y, z)$	Stress distribution function on the faces $x = \pm a$

1. INTRODUCTION

The stress distribution in rectangular prisms subjected to certain normal end forces on its two opposite faces is a fundamental problem in Applied Mechanics. Reference [1] supplies a method for calculating a symmetric end loading, assuming that the bulk stress ($\sigma_x + \sigma_y + \sigma_z$) on a lateral surface of the rectangular prism is known (i.e. measurable). Such a measurement could be obtained using advanced thermal emission techniques. The aim of this paper is to extend this concept to the case where an antisymmetric end loading is applied.

A method for calculating the bulk stresses for a given antisymmetric end loading is presented in [2]. This will be referred to as the Direct solution. In the Inverse method the end loading will be determined from a known distribution of bulk stress values on a lateral surface. In this case, bulk stresses on the lateral surface will be simulated using the Direct solution.

Figure 1 shows the orientation of the rectangular prism under consideration. The equations supplied in [2] allow calculation of the stress distribution on a lateral surface for an applied loading which is antisymmetric in y and symmetric in x and z . This paper extends this solution to calculate the stress distribution on the same lateral surface for a loading antisymmetric in z and symmetric in x and y . Inverse solutions for both loading cases have been derived. A single Inverse solution could be used to determine a loading antisymmetric in y from the stress distribution on the face $y = b$, or to determine a loading antisymmetric in z from the stress distribution on the face $z = c$. However, two separate solutions are necessary to determine the applied antisymmetric loading if only one lateral surface is available (or has been prepared) for bulk stress measurement by thermal emission.

2. DIRECT SOLUTION

The bulk stresses on the surface $y = +b$ are required for the case of a rectangular prism subjected to certain end forces. The two end loading cases considered are:

- (1) Loading antisymmetric in y , symmetric in x and z
- (2) Loading antisymmetric in z , symmetric in x and y

2.1 Loading Antisymmetric in Y

First define the Galerkin vector \mathbf{F} .

$$\mathbf{F} = iF_x + jF_y + kF_z$$

where

$$F_x = \sum_M \sum_n \frac{H_{Mn}a}{\alpha_{Mn}^3 \cosh \alpha_{Mn}a} [\alpha_{Mn}x \cosh \alpha_{Mn}x - (2\nu + \alpha_{Mn}a \coth \alpha_{Mn}a) \sinh \alpha_{Mn}x] \times \\ \times \cos \frac{n\pi z}{c} \sin \frac{M\pi y}{2b}$$

$$F_y = \sum_n \sum_l \frac{K_{nl}b}{\beta_{nl}^3 \sinh \beta_{nl}b} [\beta_{nl}y \sinh \beta_{nl}y - (2\nu + \beta_{nl}b \tanh \beta_{nl}b) \cosh \beta_{nl}y] \times \\ \times \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a}$$

$$F_z = \sum_l \sum_M \frac{L_{lM}c}{\gamma_{lM}^3 \cosh \gamma_{lM}c} [\gamma_{lM}z \cosh \gamma_{lM}z - (2\nu + \gamma_{lM}c \coth \gamma_{lM}c) \sinh \gamma_{lM}z] \times \\ \times \cos \frac{l\pi x}{a} \sin \frac{M\pi y}{2b}$$

and

$$\begin{aligned} \alpha_{Mn}^2 &= \left(\frac{M\pi}{2b}\right)^2 + \left(\frac{n\pi}{c}\right)^2 \\ \beta_{nl}^2 &= \left(\frac{n\pi}{c}\right)^2 + \left(\frac{l\pi}{a}\right)^2 \\ \gamma_{lM}^2 &= \left(\frac{l\pi}{a}\right)^2 + \left(\frac{M\pi}{2b}\right)^2 \end{aligned} \quad (1)$$

where

$$l, n = 0, 1, 2, 3, \dots \quad \text{and} \quad M = 1, 3, 5, 7, \dots$$

The stress components σ_z and τ_{yz} are then related to \mathbf{F} by the following equations.

$$\sigma_z = 2(1 - \nu) \frac{\partial}{\partial x} \nabla^2 F_x + \left(\nu \nabla^2 - \frac{\partial^2}{\partial x^2} \right) \operatorname{div} \mathbf{F}, \quad (2)$$

$$\tau_{yz} = (1 - \nu) \left(\frac{\partial}{\partial z} \nabla^2 F_y + \frac{\partial}{\partial y} \nabla^2 F_z \right) - \frac{\partial^2}{\partial y \partial z} \operatorname{div} \mathbf{F}. \quad (3)$$

This gives:

$$\begin{aligned} \sigma_z = & \sum_M \sum_n \frac{H_{Mn} a}{\cosh \alpha_{Mn} a} [(1 + \alpha_{Mn} a \coth \alpha_{Mn} a) \cosh \alpha_{Mn} x - \alpha_{Mn} x \sinh \alpha_{Mn} x] \\ & \times \sin \frac{M\pi y}{2b} \cos \frac{n\pi z}{c} + \sum_n \sum_l \frac{K_{nl} b}{\beta_{nl}^2 c^2 \sinh \beta_{nl} b} \left[2\nu n^2 \pi^2 \sinh \beta_{nl} y + \left(\frac{l\pi c}{a} \right)^2 \right. \\ & \times [(1 - \beta_{nl} b \tanh \beta_{nl} b) \sinh \beta_{nl} y + \beta_{nl} y \cosh \beta_{nl} y] \left. \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \\ & + \sum_i \sum_M \frac{L_{iM} c}{\gamma_{iM}^2 b^2 \cosh \gamma_{iM} c} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \gamma_{iM} z + \left(\frac{l\pi b}{a} \right)^2 [(1 - \gamma_{iM} c \coth \gamma_{iM} c) \right. \\ & \times \cosh \gamma_{iM} z + \gamma_{iM} z \sinh \gamma_{iM} z] \left. \right] \cos \frac{l\pi x}{a} \sin \frac{M\pi y}{2b} \end{aligned} \quad (4)$$

$$\begin{aligned} \tau_{yz} = & - \sum_M \sum_n \frac{H_{Mn} a M n \pi^2}{2 \alpha_{Mn}^2 b c \cosh \alpha_{Mn} a} \left[[1 - (2\nu + \alpha_{Mn} a \coth \alpha_{Mn} a)] \cosh \alpha_{Mn} x \right. \\ & + \alpha_{Mn} x \sinh \alpha_{Mn} x \left. \right] \cos \frac{M\pi y}{2b} \sin \frac{n\pi z}{c} + \sum_n \sum_l \frac{K_{nl} n \pi b}{\beta_{nl} c \sinh \beta_{nl} b} \left[\beta_{nl} y \sinh \beta_{nl} y \right. \\ & - \beta_{nl} b \tanh \beta_{nl} b \cosh \beta_{nl} y \left. \right] \sin \frac{n\pi z}{c} \cos \frac{l\pi x}{a} + \sum_l \sum_M \frac{L_{iM} M \pi c}{2 \gamma_{iM} b \cosh \gamma_{iM} c} \\ & \times \left[\gamma_{iM} c \coth \gamma_{iM} c \sinh \gamma_{iM} z - \gamma_{iM} z \cosh \gamma_{iM} z \right] \sin \frac{M\pi y}{2b} \cos \frac{l\pi x}{a} \end{aligned} \quad (5)$$

Expressions for $\sigma_y, \sigma_z, \tau_{xy}$ and τ_{xz} can be written in a similar manner.

With reference to Figure 1, the following boundary conditions apply.

$$\begin{aligned} \text{on } y = \pm b; \quad & \sigma_y = 0, \quad \tau_{xy} = 0 \quad \text{and} \quad \tau_{yz} = 0, \\ \text{on } z = \pm c; \quad & \sigma_z = 0, \quad \tau_{xz} = 0 \quad \text{and} \quad \tau_{yz} = 0, \\ \text{on } x = \pm a; \quad & \sigma_x = f_2(y, z), \quad \tau_{xy} = 0 \quad \text{and} \quad \tau_{xz} = 0. \end{aligned} \quad (6)$$

All the shear stress boundary conditions are exactly satisfied by (5) and similar expressions for τ_{xy} and τ_{xz} . The normal stress boundary conditions are approximately satisfied by equating low order Fourier terms in equation (4), and in similar expressions for σ_y and σ_z . Taking the double Fourier transforms of these three equations (with their approximated normal stress boundary conditions) gives the expressions (7), (8) and (9).

$$\begin{aligned} & \sum_M \frac{4H_{Mn}\delta_1 \tanh \alpha_{Mn} a (-1)^{l+(M-1)/2} \left[\nu \left(\frac{n\pi a}{c} \right)^2 (\alpha_{Mn}^2 a^2 + l^2 \pi^2) \right.}{\alpha_{Mn} b (\alpha_{Mn}^2 a^2 + l^2 \pi^2)^2} \\ & + \left. \left(\frac{M\pi a}{2b} \right)^2 l^2 \pi^2 \right] + K_{nl} \delta_2 \left(1 - \frac{2\beta_{nl} b}{\sinh 2\beta_{nl} b} \right) \\ & + \sum_M \frac{4L_{IM}\delta_3 \tanh \gamma_{IM} c (-1)^{n+(M-1)/2} \left[\nu \left(\frac{l\pi c}{a} \right)^2 (\gamma_{IM}^2 c^2 + n^2 \pi^2) \right.}{\gamma_{IM} b (\gamma_{IM}^2 c^2 + n^2 \pi^2)^2} \\ & + \left. \left(\frac{M\pi c}{2b} \right)^2 n^2 \pi^2 \right] = 0 \end{aligned} \quad (7)$$

where

$$\begin{aligned} \delta_1 &= 2 \quad \text{when } n = 0, \quad \delta_1 = 1 \quad \text{when } n \neq 0 \\ \delta_2 &= 2 \quad \text{when } n = 0 \text{ or } l = 0, \quad \delta_2 = 1 \quad \text{when } n \neq 0, \quad l \neq 0 \\ \delta_3 &= 2 \quad \text{when } l = 0, \quad \delta_3 = 1 \quad \text{when } l \neq 0 \end{aligned}$$

$$\begin{aligned} & \sum_n \frac{4H_{Mn} \tanh \alpha_{Mn} a (-1)^{l+n} \left[\nu \left(\frac{M\pi a}{2b} \right)^2 (\alpha_{Mn}^2 a^2 + l^2 \pi^2) \right.}{\alpha_{Mn} c (\alpha_{Mn}^2 a^2 + l^2 \pi^2)^2} \\ & + \left. \left(\frac{n\pi a}{c} \right)^2 l^2 \pi^2 \right] + \sum_n \frac{4K_{nl}\delta_1 \coth \beta_{nl} b (-1)^{n+(M-1)/2}}{\beta_{nl} c \left[\beta_{nl}^2 b^2 + \frac{M^2 \pi^2}{4} \right]^2} \\ & \times \left[\nu \left(\frac{l\pi b}{a} \right)^2 \left(\beta_{nl}^2 b^2 + \frac{M^2 \pi^2}{4} \right) + \left(\frac{n\pi b}{c} \right)^2 \frac{M^2 \pi^2}{4} \right] \\ & + L_{IM} \delta_1 \left(1 + \frac{2\gamma_{IM} c}{\sinh 2\gamma_{IM} c} \right) = 0 \end{aligned} \quad (8)$$

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

$$\begin{aligned}
& H_{Mn} \delta_1 \left(1 + \frac{2\alpha_{Mn} a}{\sinh 2\alpha_{Mn} a} \right) + \sum_l \frac{4K_{nl} \delta_1 \coth \beta_{nl} b (-1)^{l+(M-1)/2}}{\beta_{nl} a \left[\beta_{nl}^2 b^2 + \frac{M^2 \pi^2}{4} \right]^2} \\
& \times \left[\nu \left(\frac{n\pi b}{c} \right)^2 \left(\beta_{nl}^2 b^2 + \frac{M^2 \pi^2}{4} \right) + \left(\frac{l\pi b}{a} \right)^2 \frac{M^2 \pi^2}{4} \right] \\
& + \sum_l \frac{4L_{lm} \tanh \gamma_{lm} c (-1)^{l+n}}{\gamma_{lm} a (\gamma_{lm}^2 c^2 + n^2 \pi^2)^2} \left[\nu \left(\frac{M\pi c}{2b} \right)^2 (\gamma_{lm}^2 c^2 + n^2 \pi^2) \right. \\
& \left. + \left(\frac{l\pi c}{a} \right)^2 n^2 \pi^2 \right] = J_{Mn}
\end{aligned} \tag{9}$$

where

$$\delta_1 = 2 \quad \text{when } n = 0, \quad \delta_1 = 1 \quad \text{when } n \neq 0$$

and

$$J_{Mn} = \frac{1}{abc} \int_{-b}^{+b} \int_{-c}^{+c} f_2(y, z) \sin \frac{M\pi y}{2b} \cos \frac{n\pi z}{c} dy dz$$

Equations (7), (8) and (9) may then be solved simultaneously for the unknown Fourier coefficients. The stress at any point in the prism may then be calculated by substituting the Fourier coefficients into equations (4), (5) and similar.

Since the given equations are valid only for self equilibrating end loads, it is necessary to modify the required end loading $f(y, z)$ so that the net force and moment acting on each end face is zero. This produces a modified loading distribution $f_2(y, z)$.

$$f_2(y, z) = \left[f(y, z) - \frac{3My}{4cb^3} \right] \tag{10}$$

where M is the total moment due to $f(y, z)$, i.e.

$$M = \int_{-b}^{+b} \int_{-c}^{+c} f(y, z) y dy dz \tag{11}$$

The previous equations calculate the stresses due to the modified loading $f_2(y, z)$. The stress component σ_x , calculated from equation (4), is a Fourier series approximation to $f_2(y, z)$.

2.2 Loading Antisymmetric in Z

Again define the Galerkin vector \mathbf{F} .

$$\mathbf{F} = \mathbf{i}F_x + \mathbf{j}F_y + \mathbf{k}F_z$$

where

$$F_x = \sum_m \sum_N \frac{H_{mN}a}{\alpha_{mN}^3 \cosh \alpha_{mN}a} [\alpha_{mN}x \cosh \alpha_{mN}x - (2\nu + \alpha_{mN}a \coth \alpha_{mN}a) \sinh \alpha_{mN}x] \times \\ \times \cos \frac{m\pi y}{b} \sin \frac{N\pi z}{2c}$$

$$F_y = \sum_N \sum_l \frac{K_{Nl}b}{\beta_{Nl}^3 \cosh \beta_{Nl}b} [\beta_{Nl}y \cosh \beta_{Nl}y - (2\nu + \beta_{Nl}b \coth \beta_{Nl}b) \sinh \beta_{Nl}y] \times \\ \times \cos \frac{l\pi x}{a} \sin \frac{N\pi z}{2c}$$

$$F_z = \sum_l \sum_m \frac{L_{lm}c}{\gamma_{lm}^3 \sinh \gamma_{lm}c} [\gamma_{lm}z \sinh \gamma_{lm}z - (2\nu + \gamma_{lm}c \tanh \gamma_{lm}c) \cosh \gamma_{lm}z] \times \\ \times \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b}$$

and

$$\alpha_{mN}^2 = \left(\frac{m\pi}{b}\right)^2 + \left(\frac{N\pi}{2c}\right)^2 \\ \beta_{Nl}^2 = \left(\frac{N\pi}{2c}\right)^2 + \left(\frac{l\pi}{a}\right)^2 \\ \gamma_{lm}^2 = \left(\frac{l\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2 \quad (12)$$

where

$$l, m = 0, 1, 2, 3, \dots \quad \text{and} \quad N = 1, 3, 5, 7, \dots$$

Substituting these equations into equations (2) and (3) gives:

$$\begin{aligned}
\sigma_x = & \sum_m \sum_N \frac{H_{mNa}}{\cosh \alpha_{mNa}} [(1 + \alpha_{mNa} \coth \alpha_{mNa}) \cosh \alpha_{mNx} - \alpha_{mNx} \sinh \alpha_{mNx}] \\
& \times \cos \frac{m\pi y}{b} \sin \frac{N\pi z}{2c} + \sum_N \sum_l \frac{K_{Nl}b}{\beta_{Nl}^2 c^2 \cosh \beta_{Nl}b} \left[2\nu \frac{N^2 \pi^2}{4} \cosh \beta_{Nl}y + \left(\frac{l\pi c}{a} \right)^2 \right. \\
& \times [(1 - \beta_{Nl}b \coth \beta_{Nl}b) \cosh \beta_{Nl}y + \beta_{Nl}y \sinh \beta_{Nl}y] \left. \sin \frac{N\pi z}{2c} \cos \frac{l\pi x}{a} \right. \\
& + \sum_l \sum_m \frac{L_{lm}c}{\gamma_{lm}^2 b^2 \sinh \gamma_{lm}c} \left[2\nu m^2 \pi^2 \sinh \gamma_{lm}z + \left(\frac{l\pi b}{a} \right)^2 [(1 - \gamma_{lm}c \tanh \gamma_{lm}c) \right. \\
& \times \sinh \gamma_{lm}z + \gamma_{lm}z \cosh \gamma_{lm}z] \left. \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b} \right] \quad (13)
\end{aligned}$$

$$\begin{aligned}
\tau_{yz} = & - \sum_m \sum_N \frac{H_{mNa} m N \pi^2}{2 \alpha_{mN}^2 b c \cosh \alpha_{mNa}} \left[[1 - (2\nu + \alpha_{mN}c \coth \alpha_{mNa})] \cosh \alpha_{mNx} \right. \\
& + \alpha_{mNx} \sinh \alpha_{mNx} \left. \cos \frac{N\pi z}{2c} \sin \frac{m\pi y}{b} + \sum_N \sum_l \frac{K_{Nl} N \pi b}{2 \beta_{Nl} c \cosh \beta_{Nl}b} \left[\beta_{Nl}y \cosh \beta_{Nl}y \right. \right. \\
& - \beta_{Nl}b \coth \beta_{Nl}b \sinh \beta_{Nl}y \left. \cos \frac{N\pi z}{2c} \cos \frac{l\pi x}{a} + \sum_l \sum_m \frac{L_{lm} m \pi c}{\gamma_{lm} b \sinh \gamma_{lm}c} \right. \\
& \times \left. \left[\gamma_{lm}z \sinh \gamma_{lm}z - \gamma_{lm}c \tanh \gamma_{lm}c \cosh \gamma_{lm}z \right] \sin \frac{m\pi y}{b} \cos \frac{l\pi x}{a} \right] \quad (14)
\end{aligned}$$

Again, expressions for σ_y , σ_z , τ_{xy} and τ_{xz} can be written in a similar manner.

The boundary conditions of (6) still apply, with the exception that $f_3(y, z)$ is now used to denote the modified applied loading, i.e.

$$\begin{aligned}
& \text{on } y = \pm b; \quad \sigma_y = 0, \quad \tau_{xy} = 0 \quad \text{and} \quad \tau_{yz} = 0, \\
& \text{on } z = \pm c; \quad \sigma_z = 0, \quad \tau_{xz} = 0 \quad \text{and} \quad \tau_{yz} = 0, \\
& \text{on } x = \pm a; \quad \sigma_x = f_3(y, z), \quad \tau_{xy} = 0 \quad \text{and} \quad \tau_{xz} = 0. \quad (15)
\end{aligned}$$

Again, all the shear stress boundary conditions are automatically satisfied by (14) and similar expressions for τ_{xy} and τ_{xz} . The normal stress boundary conditions are approximately satisfied by equating low order Fourier terms in equation (13), and in similar expressions for σ_y and σ_z . Taking the double Fourier transforms of these three equations (with their approximated normal stress boundary conditions) gives the expressions (16), (17) and (18).

$$\begin{aligned} & \sum_m \frac{4H_{mN} \tanh \alpha_{mN} a (-1)^{l+m}}{\alpha_{mN} b (\alpha_{mN}^2 a^2 + l^2 \pi^2)^2} \left[\nu \left(\frac{N\pi a}{2c} \right)^2 (\alpha_{mN}^2 a^2 + l^2 \pi^2) \right. \\ & + \left(\frac{m\pi a}{b} \right)^2 l^2 \pi^2 \left. \right] + K_M \delta_1 \left(1 + \frac{2\beta_M b}{\sinh 2\beta_M b} \right) \\ & + \sum_m \frac{4L_{lm} \delta_1 \coth \gamma_{lm} c (-1)^{m+(N-1)/2}}{\gamma_{lm} b \left[\gamma_{lm}^2 c^2 + \frac{N^2 \pi^2}{4} \right]^2} \left[\nu \left(\frac{l\pi c}{a} \right)^2 \left(\gamma_{lm}^2 c^2 + \frac{N^2 \pi^2}{4} \right) \right. \\ & + \left. \left(\frac{m\pi c}{b} \right)^2 \frac{N^2 \pi^2}{4} \right] = 0 \end{aligned} \quad (16)$$

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

$$\begin{aligned} & \sum_N \frac{4H_{mN} \delta_1 \tanh \alpha_{mN} a (-1)^{l+(N-1)/2}}{\alpha_{mN} c (\alpha_{mN}^2 a^2 + l^2 \pi^2)^2} \left[\nu \left(\frac{m\pi a}{b} \right)^2 (\alpha_{mN}^2 a^2 + l^2 \pi^2) \right. \\ & + \left. \left(\frac{N\pi a}{2c} \right)^2 l^2 \pi^2 \right] + \sum_N \frac{4K_M \delta_2 \tanh \beta_M b (-1)^{m+(N-1)/2}}{\beta_M c (\beta_M^2 b^2 + m^2 \pi^2)^2} \\ & \times \left[\nu \left(\frac{l\pi b}{a} \right)^2 (\beta_M^2 b^2 + m^2 \pi^2) + \left(\frac{N\pi b}{2c} \right)^2 m^2 \pi^2 \right] \\ & + L_{lm} \delta_3 \left(1 - \frac{2\gamma_{lm} c}{\sinh 2\gamma_{lm} c} \right) = 0 \end{aligned} \quad (17)$$

where

$$\begin{aligned} \delta_1 &= 2 \quad \text{when } m = 0, \quad \delta_1 = 1 \quad \text{when } m \neq 0 \\ \delta_2 &= 2 \quad \text{when } l = 0, \quad \delta_2 = 1 \quad \text{when } l \neq 0 \\ \delta_3 &= 2 \quad \text{when } m = 0 \text{ or } l = 0, \quad \delta_3 = 1 \quad \text{when } m \neq 0, \quad l \neq 0 \end{aligned}$$

$$\begin{aligned}
& H_{mN} \delta_1 \left(1 + \frac{2\alpha_{mN}a}{\sinh 2\alpha_{mN}a} \right) + \sum_l \frac{4K_N \tanh \beta_N b (-1)^{l+m}}{\beta_N a (\beta_N^2 b^2 + m^2 \pi^2)^2} \\
& \times \left[\nu \left(\frac{N\pi b}{2c} \right)^2 (\beta_N^2 b^2 + m^2 \pi^2) + \left(\frac{l\pi b}{a} \right)^2 m^2 \pi^2 \right] \\
& + \sum_l \frac{4L_{lm} \delta_1 \coth \gamma_{lm} c (-1)^{l+(N-1)/2}}{\gamma_{lm} a [\gamma_{lm}^2 c^2 + \frac{N^2 \pi^2}{4}]^2} \left[\nu \left(\frac{m\pi c}{b} \right)^2 \left(\gamma_{lm}^2 c^2 + \frac{N^2 \pi^2}{4} \right) \right. \\
& \left. + \left(\frac{l\pi c}{a} \right)^2 \frac{N^2 \pi^2}{4} \right] = J_{mN}
\end{aligned} \tag{18}$$

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

and

$$J_{mN} = \frac{1}{abc} \int_{-b}^{+b} \int_{-c}^{+c} f_3(y, z) \cos \frac{m\pi y}{b} \sin \frac{N\pi z}{2c} dy dz$$

Equations (16), (17) and (18) may then be solved simultaneously for the unknown Fourier coefficients. The stress at any point in the prism may then be calculated by substituting the Fourier coefficients into equations (13), (14) and similar.

The required end loading $f(y, z)$ must again be modified so that the net force and moment acting on each end face is zero. This produces a modified loading distribution $f_3(y, z)$.

$$f_3(y, z) = \left[f(y, z) - \frac{3Mz}{4bc^3} \right] \tag{19}$$

where M is the total moment due to $f(y, z)$, i.e.

$$M = \int_{-b}^{+b} \int_{-c}^{+c} f(y, z) z dy dz \tag{20}$$

The previous equations calculate the stresses due to the modified loading $f_3(y, z)$. The stress component σ_x , calculated from equation (13), is a Fourier series approximation to $f_3(y, z)$.

3. INVERSE SOLUTION

The main aim of this section is to provide an Inverse solution to the problem, which involves calculation of the end stress σ_x from the known bulk stress distribution ($\sigma_x + \sigma_y + \sigma_z$) on one lateral surface of the prism. Consider the case when we know the bulk stress distribution on the surface $y = +b$. On this surface, $\sigma_y = 0$, and so the bulk stress is $\sigma_x + \sigma_z$. As for the Direct solution, two loading cases are considered, i.e.

- (1) Loading antisymmetric in y , symmetric in x and z
- (2) Loading antisymmetric in z , symmetric in x and y

3.1 Loading Antisymmetric in Y

By summing equation (4), and a similar equation for σ_z , the following expression for the bulk stress is obtained.

$$\begin{aligned}
 \sigma_b = & \sum_M \sum_n \frac{H_{Mn}a}{\cosh \alpha_{Mn}a} \left\{ (1 + \alpha_{Mn}a \coth \alpha_{Mn}a) \cosh \alpha_{Mn}x - \alpha_{Mn}x \sinh \alpha_{Mn}x \right\} \\
 & \times \sin \frac{M\pi y}{2b} \cos \frac{n\pi z}{c} + \sum_n \sum_l \frac{K_{nl}b}{\beta_{nl}^2 c^2 \sinh \beta_{nl}b} \left[2\nu n^2 \pi^2 \sinh \beta_{nl}y + \left(\frac{l\pi c}{a} \right)^2 \right. \\
 & \times \left. \left\{ (1 - \beta_{nl}b \tanh \beta_{nl}b) \sinh \beta_{nl}y + \beta_{nl}y \cosh \beta_{nl}y \right\} \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \\
 & + \sum_l \sum_M \frac{L_{lM}c}{\gamma_{lM}^2 b^2 \cosh \gamma_{lM}c} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \gamma_{lM}z + \left(\frac{l\pi b}{a} \right)^2 \left\{ (1 - \gamma_{lM}c \coth \gamma_{lM}c) \right. \right. \\
 & \times \left. \left. \cosh \gamma_{lM}z + \gamma_{lM}z \sinh \gamma_{lM}z \right\} \right] \cos \frac{l\pi x}{a} \sin \frac{M\pi y}{2b} \\
 & + \sum_l \sum_M \frac{L_{lM}c}{\cosh \gamma_{lM}c} \left\{ (1 + \gamma_{lM}c \coth \gamma_{lM}c) \cosh \gamma_{lM}z - \gamma_{lM}z \sinh \gamma_{lM}z \right\} \\
 & \times \cos \frac{l\pi x}{a} \sin \frac{M\pi y}{2b} + \sum_M \sum_n \frac{H_{Mn}a}{\alpha_{Mn}^2 b^2 \cosh \alpha_{Mn}a} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \alpha_{Mn}x + \left(\frac{n\pi b}{c} \right)^2 \right. \\
 & \times \left. \left\{ (1 - \alpha_{Mn}a \coth \alpha_{Mn}a) \cosh \alpha_{Mn}x + \alpha_{Mn}x \sinh \alpha_{Mn}x \right\} \right] \sin \frac{M\pi y}{2b} \cos \frac{n\pi z}{c} \\
 & + \sum_n \sum_l \frac{K_{nl}b}{\beta_{nl}^2 a^2 \sinh \beta_{nl}b} \left[2\nu l^2 \pi^2 \sinh \beta_{nl}y + \left(\frac{n\pi a}{c} \right)^2 \left\{ (1 - \beta_{nl}b \tanh \beta_{nl}b) \right. \right. \\
 & \times \left. \left. \sinh \beta_{nl}y + \beta_{nl}y \cosh \beta_{nl}y \right\} \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \quad (21)
 \end{aligned}$$

By taking the Fourier transform of this equation, as shown in Appendix A, the following results are obtained.

$$\begin{aligned}
& \frac{1}{abc} \int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z)|_{y=b} \cos \frac{l\pi x}{a} \cos \frac{n\pi z}{2c} dx dz \\
&= \sum_M \frac{4H_{Mn}\delta_1(-1)^{l+(m-1)/2}}{b[\alpha_{Mn}^2 + (\frac{l\pi}{a})^2]^2} \left[\alpha_{Mn}^3 + \frac{1}{\alpha_{Mn}b^2} \left\{ \nu \frac{M^2\pi^2}{4} \left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a} \right)^2 \right] \right. \right. \\
&\quad \left. \left. + \left(\frac{n\pi b}{c} \right)^2 \left(\frac{l\pi}{a} \right)^2 \right\} \right] \tanh \alpha_{Mn}a \\
&+ \sum_M \frac{4\delta_3 L_{lM}(-1)^{n+(M-1)/2}}{b[\gamma_{lM}^2 + (\frac{n\pi}{c})^2]^2} \left[\gamma_{lM}^3 + \frac{1}{\gamma_{lM}b^2} \left\{ \nu \frac{M^2\pi^2}{4} \left[\gamma_{lM}^2 + \left(\frac{n\pi}{c} \right)^2 \right] \right. \right. \\
&\quad \left. \left. + \left(\frac{l\pi b}{a} \right)^2 \left(\frac{n\pi}{c} \right)^2 \right\} \right] \tanh \gamma_{lM}c \\
&+ \frac{K_{nl}\delta_2\pi^2}{\beta_{nl}^2 a^2 c^2} (a^2 n^2 + c^2 l^2) (2\nu + 1 - \beta_{nl}b \tanh \beta_{nl}b + \beta_{nl}b \coth \beta_{nl}b) \quad (22)
\end{aligned}$$

where

$$\begin{aligned}
\delta_1 &= 2 \quad \text{when } n = 0, \quad \delta_1 = 1 \quad \text{when } n \neq 0 \\
\delta_2 &= 2 \quad \text{when } n = 0 \text{ or } l = 0, \quad \delta_2 = 1 \quad \text{when } n \neq 0, \quad l \neq 0 \\
\delta_3 &= 2 \quad \text{when } l = 0, \quad \delta_3 = 1 \quad \text{when } l \neq 0
\end{aligned}$$

Equation (22) may then be applied for each value of n and l to give a set of linear simultaneous equations with the Fourier coefficients H_{Mn} , K_{nl} and L_{lM} as the unknowns. Since the boundary conditions of equation (6) still apply, equations (7) and (8) are also used in conjunction with (22). In effect, equation (22) replaces equation (9) for the Inverse solution. This imposes the restriction that $l = (M - 1)/2$, so that the number of linear equations produced by (7) (8) and (22) equals the number of unknown Fourier coefficients. The resulting set of equations is then solved to produce a set of Fourier coefficients H_{Mn} , K_{nl} and L_{lM} for the Inverse problem. Equation (4) is then used to obtain the applied stress σ_x .

3.2 Loading Antisymmetric in Z

By summing equation (13), and a similar equation for σ_z , the following expression for the bulk stress is obtained.

$$\begin{aligned}
 \sigma_b = & \sum_m \sum_N \frac{H_{mNa}}{\cosh \alpha_{mNa}} \left\{ (1 + \alpha_{mNa} \coth \alpha_{mNa}) \cosh \alpha_{mNx} - \alpha_{mNx} \sinh \alpha_{mNx} \right\} \\
 & \times \cos \frac{m\pi y}{b} \sin \frac{N\pi z}{2c} + \sum_N \sum_l \frac{K_{Nb}}{\beta_{Nl}^2 c^2 \cosh \beta_{Nl} b} \left[2\nu \frac{N^2 \pi^2}{4} \cosh \beta_{Nl} y + \left(\frac{l\pi c}{a} \right)^2 \right. \\
 & \times \left. \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) \cosh \beta_{Nl} y + \beta_{Nl} y \sinh \beta_{Nl} y \right\} \right] \sin \frac{N\pi z}{2c} \cos \frac{l\pi x}{a} \\
 & + \sum_l \sum_m \frac{L_{lm} c}{\gamma_{lm}^2 b^2 \sinh \gamma_{lm} c} \left[2\nu m^2 \pi^2 \sinh \gamma_{lm} z + \left(\frac{l\pi b}{a} \right)^2 \left\{ (1 - \gamma_{lm} c \tanh \gamma_{lm} c) \right. \right. \\
 & \times \left. \left. \sinh \gamma_{lm} z + \gamma_{lm} z \cosh \gamma_{lm} z \right\} \right] \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b} \\
 & + \sum_l \sum_m \frac{L_{lm} c}{\sinh \gamma_{lm} c} \left\{ (1 + \gamma_{lm} c \tanh \gamma_{lm} c) \sinh \gamma_{lm} z - \gamma_{lm} z \cosh \gamma_{lm} z \right\} \\
 & \times \cos \frac{l\pi x}{a} \cos \frac{m\pi y}{b} + \sum_m \sum_N \frac{H_{mNa}}{\alpha_{mN}^2 b^2 \cosh \alpha_{mNa}} \left[2\nu m^2 \pi^2 \cosh \alpha_{mNx} + \left(\frac{N\pi b}{2c} \right)^2 \right. \\
 & \times \left. \left\{ (1 - \alpha_{mNa} \coth \alpha_{mNa}) \cosh \alpha_{mNx} + \alpha_{mNx} \sinh \alpha_{mNx} \right\} \right] \cos \frac{m\pi y}{b} \sin \frac{N\pi z}{2c} \\
 & + \sum_N \sum_l \frac{K_{Nb}}{\beta_{Nl}^2 a^2 \cosh \beta_{Nl} b} \left[2\nu l^2 \pi^2 \cosh \beta_{Nl} y + \left(\frac{N\pi a}{2c} \right)^2 \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) \right. \right. \\
 & \times \left. \left. \cosh \beta_{Nl} y + \beta_{Nl} y \sinh \beta_{Nl} y \right\} \right] \sin \frac{N\pi z}{2c} \cos \frac{l\pi x}{a}
 \end{aligned} \tag{23}$$

By taking the Fourier transform of this equation, as shown in Appendix B, the following results are obtained.

$$\begin{aligned}
& \frac{1}{abc} \int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z)|_{y=b} \cos \frac{l\pi x}{a} \sin \frac{N\pi z}{2c} dx dz \\
&= \sum_m \frac{4H_{mN}(-1)^{m+l}}{b \left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right]^2} \left[\alpha_{mN}^3 + \frac{1}{\alpha_{mN} b^2} \left\{ \nu m^2 \pi^2 \left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right] \right. \right. \right. \\
&\quad \left. \left. \left. + \left(\frac{N\pi b}{2c} \right)^2 \left(\frac{l\pi}{a} \right)^2 \right\} \right] \tanh \alpha_{mN} a \\
&+ \sum_m \frac{4\delta_1 L_{lm}(-1)^{m+(N-1)/2}}{b \left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right]^2} \left[\gamma_{lm}^3 + \frac{1}{\gamma_{lm} b^2} \left\{ \nu m^2 \pi^2 \left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right] \right. \right. \right. \\
&\quad \left. \left. \left. + \left(\frac{l\pi b}{a} \right)^2 \left(\frac{N\pi}{2c} \right)^2 \right\} \right] \coth \gamma_{lm} c \\
&+ \frac{K_N \delta_1 \pi^2}{\beta_N^2 a^2 c^2} \left[\frac{N^2 a^2}{4} + c^2 l^2 \right] (2\nu + 1 - \beta_N b \coth \beta_N b + \beta_N b \tanh \beta_N b) \quad (24)
\end{aligned}$$

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

As for the case of an applied loading antisymmetric in y , a set of Fourier coefficients H_{mN} , K_N and L_{lm} may be found by solving the set of simultaneous equations produced by (16), (17) and (24). Equation (13) is then used to obtain the applied stress σ_x .

4. RESULTS AND DISCUSSION

The Direct and Inverse programs were implemented using FORTRAN 77 on an Apollo DN10000 computer. The Gaussian Quadrature method was used for the integrations required by equations (9),(18),(22) and (24). The resulting sets of linear simultaneous equations were solved using Gaussian elimination with full pivoting. Double precision was used for all calculations.

4.1 Direct Results

The equations given in the previous sections are applicable to a prism with an arbitrary rectangular cross section. However, for illustrative purposes, a square cross section with a square loading area was considered, where $b = c = 2$ and $k_1 = k_2 = 0.5$. First consider a loading symmetric in x and y , and antisymmetric in z . With reference to Figure 3, a stress of $f(y, z) = \pm 16$ was applied inside the loading area, with $f(y, z) = 0$ outside the loaded area. Equations (19) and (20) must first be used to produce a self-equilibrating loading $f_3(y, z)$ on the end face. From (20) the total moment is:

$$\begin{aligned} M &= \int_{-c}^{+c} \int_{-b}^{+b} f(y, z) z \, dy \, dz \\ &= \int_{-k_1 b}^{+k_1 b} \left[\int_{-k_2 c}^0 -16z \, dz + \int_0^{+k_2 c} 16z \, dz \right] dy + 0 \\ &= 16k_2^2 c^2 2k_1 b \\ &= 32k_2^2 k_1 b c^2 \end{aligned}$$

Applying (19) gives the modified loading function.

$$\begin{aligned} f_3(y, z) &= \left[f(y, z) - \frac{3Mz}{4bc^3} \right] \\ &= f(y, z) - \frac{24k_2^2 k_1 z}{c} \\ &= f(y, z) - 1.5z \end{aligned}$$

Hence,

$$\begin{aligned} f_3(y, z) &= -1.5z \text{ outside the loading area} \\ &= +16 - 1.5z \text{ inside the loading area, } 0 \leq z \leq k_2 c \\ &= -16 - 1.5z \text{ inside the loading area, } -k_2 c \leq z \leq 0 \end{aligned}$$

Sample results from the Direct method for an applied loading antisymmetric in z are shown in Figure 4. Plots of the bulk stress distribution on the $y = +b$ surface and the end stress σ_x are presented. These results were obtained using seven terms of the Fourier series (i.e. $l, m = 0, 1, \dots, 7$ and $N = 1, 3, \dots, 15$), which showed good convergence. Shown in Figure 4a is the surface plot of σ_x , with its corresponding values indicated in the contour plot, Figure 4b. The plots show the stress values ranging from -18.0 to $+18.0$, in good agreement with the applied stress field $f_3(y, z)$. The bulk stress distribution σ_b on the $y = +b$ surface is shown in Figures 4c and 4d.

Similar results are shown in Figure 5 for an applied loading which is antisymmetric in y , but symmetric in x and z . The modified loading function $f_2(y, z)$, produced by equations (10) and (11) is shown in Figure 2 as:

$$\begin{aligned} f_2(y, z) &= -1.5y \text{ outside the loading area} \\ &= +16 - 1.5y \text{ inside the loading area, } 0 \leq y \leq k_1 b \\ &= -16 - 1.5y \text{ inside the loading area, } -k_1 b \leq y \leq 0 \end{aligned}$$

4.2 Inverse Results

In the Inverse problem, the bulk stress field, generated by the Direct method, is used to calculate the applied end stress. For comparison, the results of the stress distribution σ_x obtained from both the Direct and Inverse methods, at different Fourier indices, are presented. Figure 6 shows the results obtained for an applied loading antisymmetric in z , symmetric in x and y . Figure 7 shows similar results for an applied loading antisymmetric in y , symmetric in x and z .

Reference [1] presented similar results for the stress distribution for a prism with symmetric end loads. It also demonstrated that more Gauss points are needed for numeric integration to obtain a reasonable Inverse solution for a high number of Fourier coefficients. The antisymmetric solution showed a similar trend. The results presented in Figures 6 and 7 use the minimum number of Gauss points to obtain a reasonable solution. Further increases in the number of Fourier coefficients and Gauss points were not warranted for preliminary work due to the large increase in computer resources required.

5. CONCLUSION

This work has successfully extended previous work on the stress distribution in rectangular prisms to include the case of antisymmetric end loading. This distribution may be calculated from a set of bulk stress measurements on one lateral surface of the prism. Analysis has been performed for the case where the bulk stress distribution used in the Inverse method was produced by the Direct method.

Although the concept has been successfully proven, a considerable amount of work will be required to extend this methodology to a procedure which can be routinely used for the analysis of thermoelastic measurements from real structures.

REFERENCES

- [1] Truong, T. and Bennett, J. "Determination of Symmetric End Loads from Bulk Stresses on a Rectangular Prism", ARL-STRUC-TM-536, Aeronautical Research Laboratory, Melbourne, Australia (1991).
- [2] Sundara Raja Iyengar, K.T. and Prabhakara, M.K. "A Three Dimensional Elasticity Solution for Rectangular Prism Under End Loads", Zeitschrift für Angewandte Mathematik und Mechanik, Vol. 49, June, 1969, pp. 321-332.

ACKNOWLEDGEMENTS

The author would like to thank Dr. Ton Tran-Cong and Dr. Rhys Jones for their direction of this project.

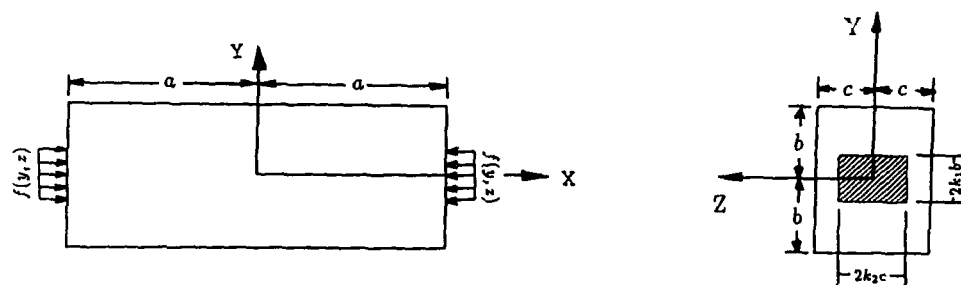


FIG. 1 RECTANGULAR PRISM WITH CENTRAL LOADING

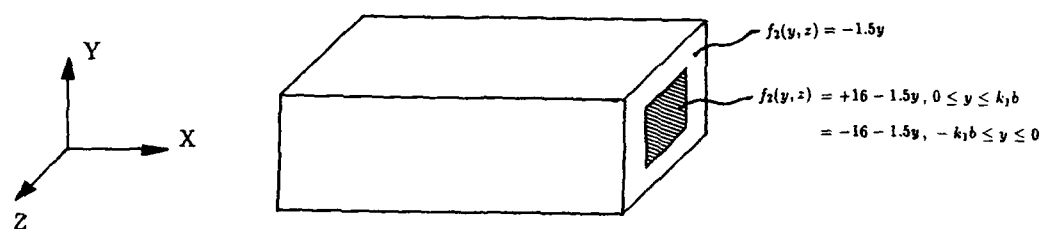


FIG. 2 APPLIED LOADING ANTISYMMETRIC IN Y

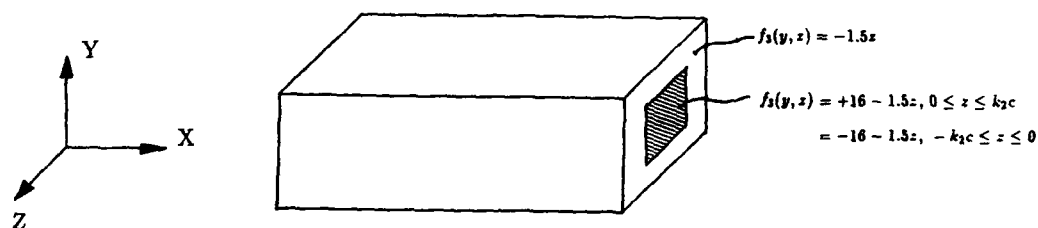


FIG. 3 APPLIED LOADING ANTISYMMETRIC IN Z

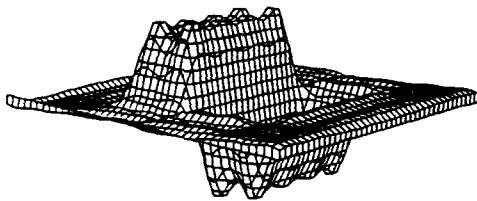


Fig. 4a Applied End Loading

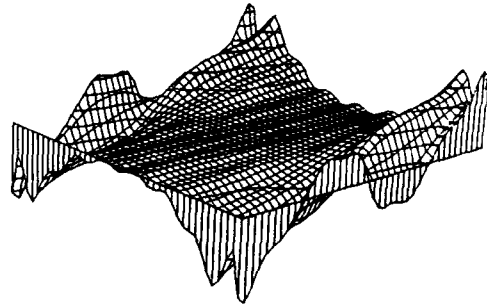


Fig. 4c Bulk Stress on $y = b$

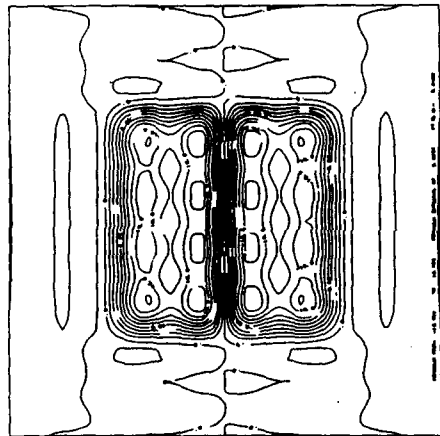


Fig. 4b Applied End Loading

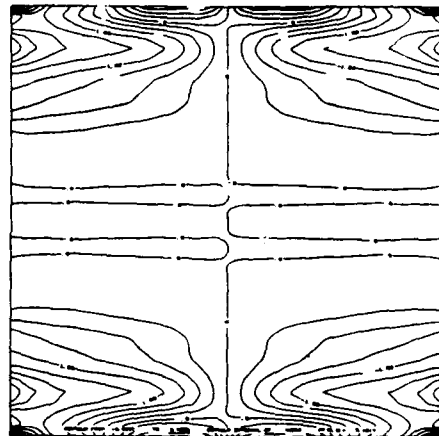


Fig. 4d Bulk Stress on $y = b$

FIG. 4 DIRECT SOLUTION FOR APPLIED LOADING
ANTISYMMETRIC IN Z

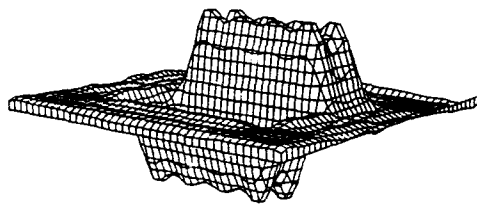


Fig. 5a Applied End Loading

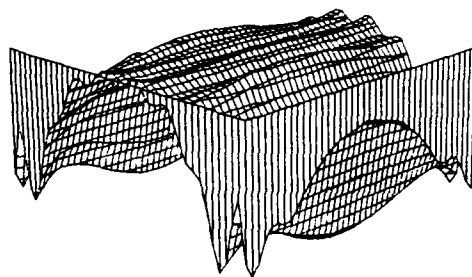


Fig. 5c Bulk Stress on $y = b$

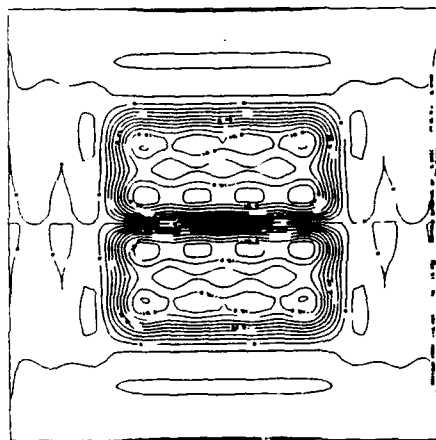


Fig. 5b Applied End Loading

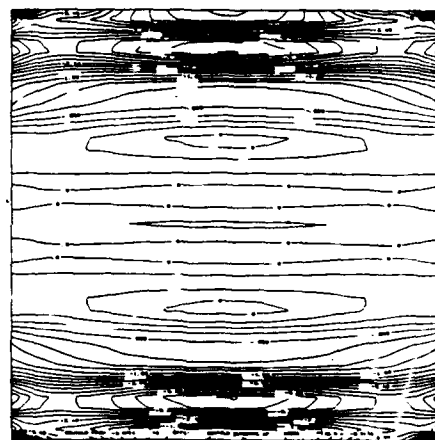
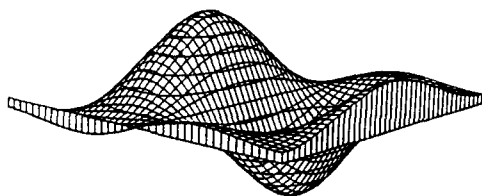
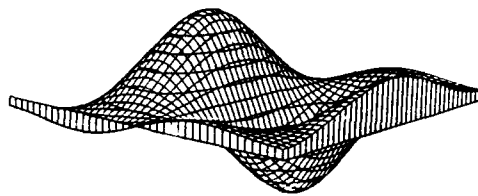


Fig. 5d Bulk Stress on $y = b$

FIG. 5 DIRECT SOLUTION FOR APPLIED LOADING:
ANTISYMMETRIC IN Y

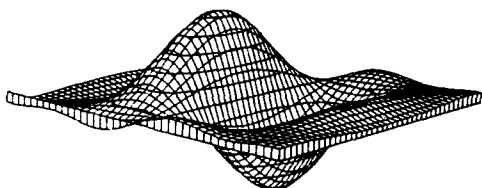


Direct Solution

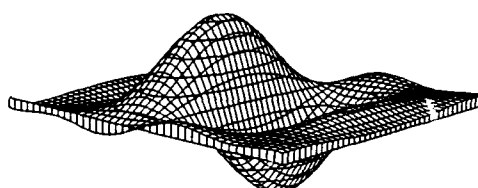


Inverse Solution

Fig 6a. $l, m = 0, 1; N = 1, 3$. Two point integration.

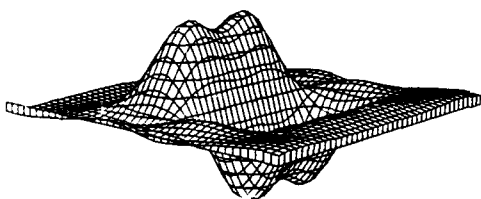


Direct Solution

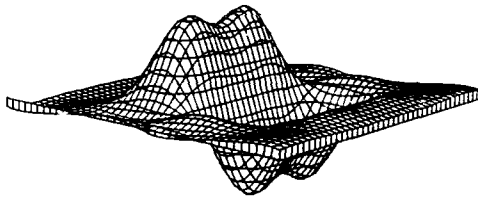


Inverse Solution

Fig 6b. $l, m = 0, 1, 2; N = 1, 3, 5$. Two point integration.

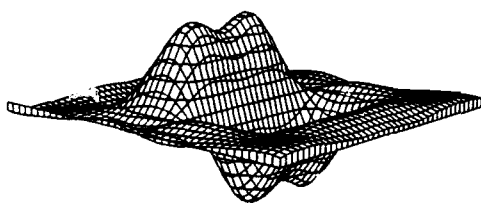


Direct Solution

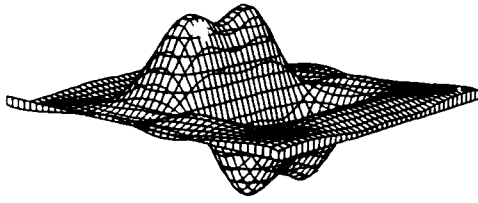


Inverse Solution

Fig 6c. $l, m = 0, 1...3; N = 1, 3...7$. Three point integration.

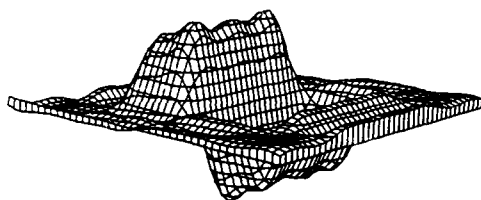


Direct Solution

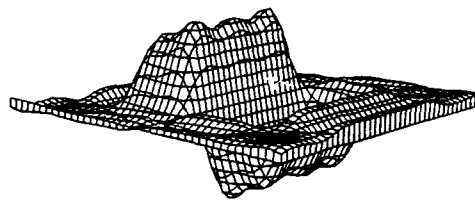


Inverse Solution

Fig 6d. $l, m = 0, 1...4; N = 1, 3...9$. Four point integration.

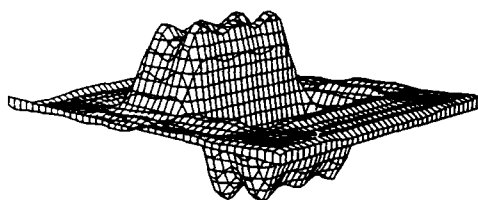


Direct Solution

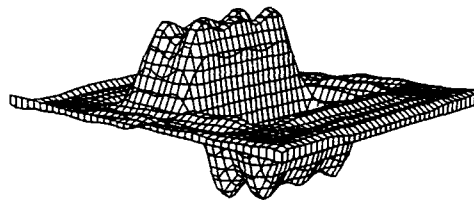


Inverse Solution

Fig 6e. $l, m = 0, 1 \dots 5$; $N = 1, 3 \dots 11$. Four point integration.

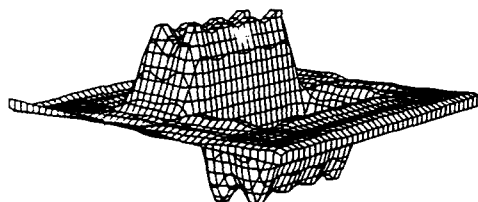


Direct Solution

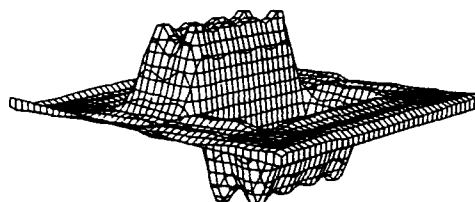


Inverse Solution

Fig 6f. $l, m = 0, 1 \dots 6$; $N = 1, 3 \dots 13$. Five point integration.



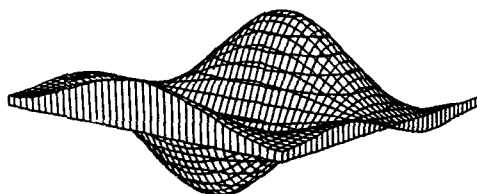
Direct Solution



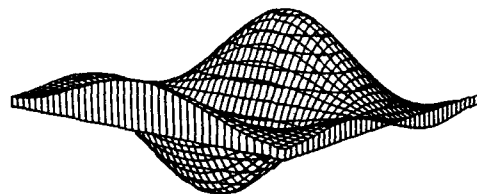
Inverse Solution

Fig 6g. $l, m = 0, 1 \dots 7$; $N = 1, 3 \dots 15$. Five point integration.

FIG. 6 DIRECT AND INVERSE SOLUTIONS FOR APPLIED
LOADING ANTISYMMETRIC IN Z

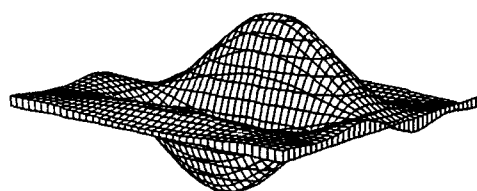


Direct Solution

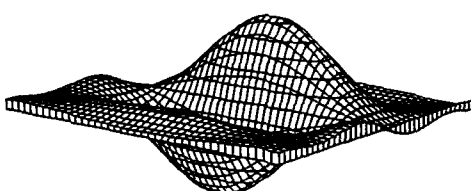


Inverse Solution

Fig 7a. $l, n = 0, 1; M = 1, 3$. Two point integration.

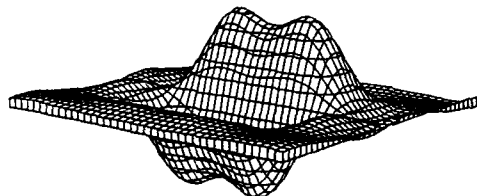


Direct Solution

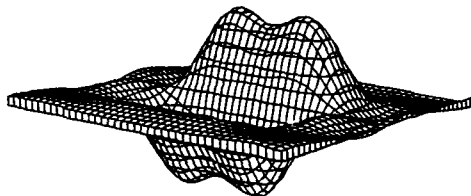


Inverse Solution

Fig 7b. $l, n = 0, 1, 2; M = 1, 3, 5$. Two point integration.

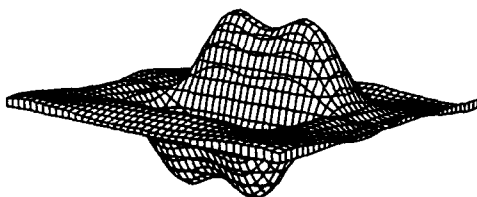


Direct Solution

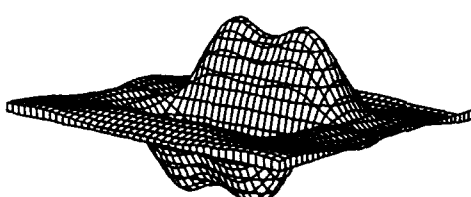


Inverse Solution

Fig 7c. $l, n = 0, 1...3; M = 1, 3...7$. Three point integration.

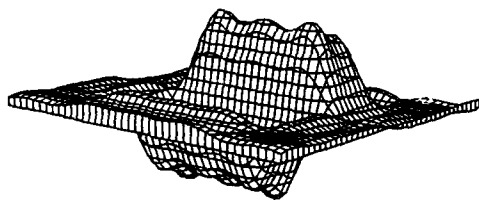


Direct Solution

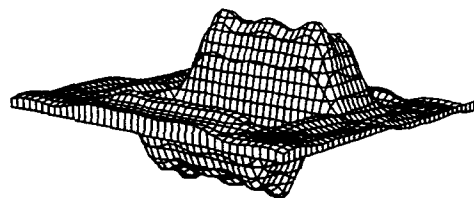


Inverse Solution

Fig 7d. $l, n = 0, 1...4; M = 1, 3...9$. Four point integration.

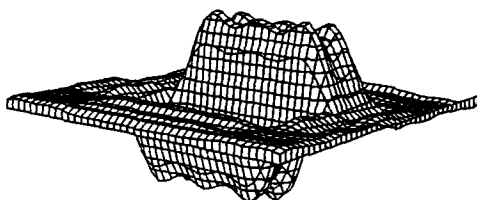


Direct Solution

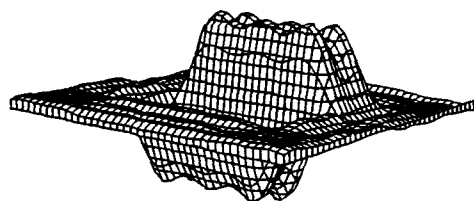


Inverse Solution

Fig 7e. $l, n = 0, 1...5; M = 1, 3...11$. Four point integration.

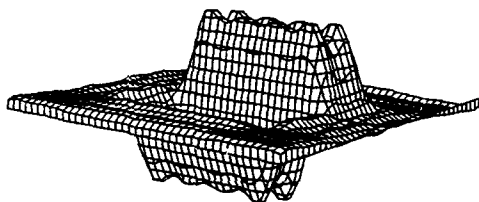


Direct Solution

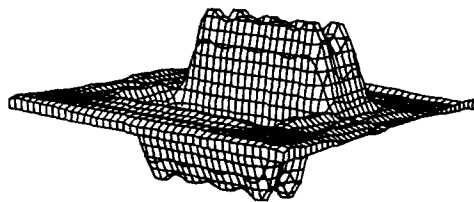


Inverse Solution

Fig 7f. $l, n = 0, 1...6; M = 1, 3...13$. Five point integration.



Direct Solution



Inverse Solution

Fig 7g. $l, n = 0, 1...7; M = 1, 3...15$. Five point integration.

FIG. 7 DIRECT AND INVERSE SOLUTIONS FOR APPLIED LOADING ANTISYMMETRIC IN Y

DERIVATION OF EQUATION (22)

From Equation (21)

$$\begin{aligned}
\sigma_b = & \sum_M \sum_n \frac{H_{Mn} a}{\cosh \alpha_{Mn} a} \left\{ (1 + \alpha_{Mn} a \coth \alpha_{Mn} a) \cosh \alpha_{Mn} x - \alpha_{Mn} x \sinh \alpha_{Mn} x \right\} \\
& \times \sin \frac{M \pi y}{2b} \cos \frac{n \pi z}{c} + \sum_n \sum_l \frac{K_{nl} b}{\beta_{nl}^2 c^2 \sinh \beta_{nl} b} \left[2\nu n^2 \pi^2 \sinh \beta_{nl} y + \left(\frac{l \pi c}{a} \right)^2 \right. \\
& \times \left. \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) \sinh \beta_{nl} y + \beta_{nl} y \cosh \beta_{nl} y \right\} \right] \cos \frac{n \pi z}{c} \cos \frac{l \pi x}{a} \\
& + \sum_l \sum_M \frac{L_{lM} c}{\gamma_{lM}^2 b^2 \cosh \gamma_{lM} c} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \gamma_{lM} z + \left(\frac{l \pi b}{a} \right)^2 \left\{ (1 - \gamma_{lM} c \coth \gamma_{lM} c) \right. \right. \\
& \times \left. \left. \cosh \gamma_{lM} z + \gamma_{lM} z \sinh \gamma_{lM} z \right\} \right] \cos \frac{l \pi x}{a} \sin \frac{M \pi y}{2b} \\
& + \sum_l \sum_M \frac{L_{lM} c}{\cosh \gamma_{lM} c} \left\{ (1 + \gamma_{lM} c \coth \gamma_{lM} c) \cosh \gamma_{lM} z - \gamma_{lM} z \sinh \gamma_{lM} z \right\} \\
& \times \cos \frac{l \pi x}{a} \sin \frac{M \pi y}{2b} + \sum_M \sum_n \frac{H_{Mn} a}{\alpha_{Mn}^2 b^2 \cosh \alpha_{Mn} a} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \alpha_{Mn} x + \left(\frac{n \pi b}{c} \right)^2 \right. \\
& \times \left. \left\{ (1 - \alpha_{Mn} a \coth \alpha_{Mn} a) \cosh \alpha_{Mn} x + \alpha_{Mn} x \sinh \alpha_{Mn} x \right\} \right] \sin \frac{M \pi y}{2b} \cos \frac{n \pi z}{c} \\
& + \sum_n \sum_l \frac{K_{nl} b}{\beta_{nl}^2 a^2 \sinh \beta_{nl} b} \left[2\nu l^2 \pi^2 \sinh \beta_{nl} y + \left(\frac{n \pi a}{c} \right)^2 \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) \right. \right. \\
& \times \left. \left. \sinh \beta_{nl} y + \beta_{nl} y \cosh \beta_{nl} y \right\} \right] \cos \frac{n \pi z}{c} \cos \frac{l \pi x}{a}
\end{aligned}$$

On a lateral surface of $y = b$, $\sigma_y = 0$ and so the bulk stress on this surface becomes $\sigma_b = \sigma_x + \sigma_z$. Putting $y = b$ into the above equation, and noting that $\sin \frac{M \pi}{2} = (-1)^{(M-1)/2}$ for $M = 1, 3, 5, \dots$, yields the following result.

APPENDIX A

$$\sigma_y = \sum_M \sum_n \frac{H_{Mn} a (-1)^{(M-1)/2}}{\cosh \alpha_{Mn} a} \left\{ (1 + \alpha_{Mn} a \coth \alpha_{Mn} a) \cosh \alpha_{Mn} x - \alpha_{Mn} x \sinh \alpha_{Mn} x \right\} \cos \frac{n\pi z}{c} \quad (I)$$

$$+ \sum_n \sum_l \frac{K_{nl} b}{\beta_{nl}^2 c^2 \sinh \beta_{nl} b} \left[2\nu n^2 \pi^2 \sinh \beta_{nl} b + \left(\frac{l\pi c}{a} \right)^2 \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) \right. \right. \\ \left. \left. \times \sinh \beta_{nl} b + \beta_{nl} b \cosh \beta_{nl} b \right\} \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \quad (II)$$

$$+ \sum_l \sum_M \frac{L_{lM} c (-1)^{(M-1)/2}}{\gamma_{lM}^2 b^2 \cosh \gamma_{lM} c} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \gamma_{lM} z + \left(\frac{l\pi b}{a} \right)^2 \left\{ (1 - \gamma_{lM} c \coth \gamma_{lM} c) \right. \right. \\ \left. \left. \times \cosh \gamma_{lM} z + \gamma_{lM} z \sinh \gamma_{lM} z \right\} \right] \cos \frac{l\pi x}{a} \quad (III)$$

$$+ \sum_l \sum_M \frac{L_{lM} c (-1)^{(M-1)/2}}{\cosh \gamma_{lM} c} \left\{ (1 + \gamma_{lM} c \coth \gamma_{lM} c) \cosh \gamma_{lM} z - \gamma_{lM} z \sinh \gamma_{lM} z \right\} \cos \frac{l\pi x}{a} \quad (IV)$$

$$+ \sum_M \sum_n \frac{H_{Mn} a (-1)^{(M-1)/2}}{\alpha_{Mn}^2 b^2 \cosh \alpha_{Mn} a} \left[2\nu \frac{M^2 \pi^2}{4} \cosh \alpha_{Mn} x + \left(\frac{n\pi b}{c} \right)^2 \left\{ (1 - \alpha_{Mn} a \coth \alpha_{Mn} a) \right. \right. \\ \left. \left. \times \cosh \alpha_{Mn} x + \alpha_{Mn} x \sinh \alpha_{Mn} x \right\} \right] \cos \frac{n\pi z}{c} \quad (V)$$

$$+ \sum_n \sum_l \frac{K_{nl} b}{\beta_{nl}^2 a^2 \sinh \beta_{nl} b} \left[2\nu l^2 \pi^2 \sinh \beta_{nl} b + \left(\frac{n\pi a}{c} \right)^2 \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) \right. \right. \\ \left. \left. \times \sinh \beta_{nl} b + \beta_{nl} b \cosh \beta_{nl} b \right\} \right] \cos \frac{n\pi z}{c} \cos \frac{l\pi x}{a} \quad (VI)$$

To determine the Fourier coefficients H_{Mn} , K_{nl} and L_{lM} it is necessary to take the double Fourier transform of the above equation.

$$\int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\ = \int_{-c}^{+c} \int_{-a}^{+a} \left[(I) + (II) + (III) + (IV) + (V) + (VI) \right] \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz$$

Note that p and q are now fixed positive integers.

APPENDIX A

Integrating the RHS, taking term (I)

$$\begin{aligned}
 & \int_{-c}^{+c} \int_{-a}^{+a} (I) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\
 &= \sum_M \sum_n \frac{H_{Mn} a (-1)^{(M-1)/2}}{\cosh \alpha_{Mn} a} \left[(1 + \alpha_{Mn} a \coth \alpha_{Mn} a) \int_{-a}^{+a} \cosh \alpha_{Mn} x \cos \frac{p\pi x}{a} dx \right. \\
 &\quad \left. - \alpha_{Mn} \int_{-a}^{+a} x \sinh \alpha_{Mn} x \cos \frac{p\pi x}{a} dx \right] \int_{-c}^{+c} \cos \frac{n\pi z}{c} \cos \frac{q\pi z}{c} dz \\
 &= \sum_M \frac{H_{Mn} a c \delta_1 (-1)^{(M-1)/2}}{\cosh \alpha_{Mn} a} \left[(1 + \alpha_{Mn} a \coth \alpha_{Mn} a) \frac{2\alpha_{Mn} (-1)^p}{\alpha_{Mn}^2 + \left(\frac{p\pi}{a}\right)^2} \sinh \alpha_{Mn} a \right. \\
 &\quad \left. - \frac{2\alpha_{Mn}^2 a (-1)^p}{\alpha_{Mn}^2 + \left(\frac{p\pi}{a}\right)^2} \cosh \alpha_{Mn} a + \frac{2\alpha_{Mn} (-1)^p \left[\alpha_{Mn}^2 - \left(\frac{p\pi}{a}\right)^2\right]}{\left[\alpha_{Mn}^2 + \left(\frac{p\pi}{a}\right)^2\right]^2} \sinh \alpha_{Mn} a \right] \\
 &= \sum_M \frac{2H_{Mn} \alpha_{Mn} \delta_1 a c (-1)^{p+(M-1)/2}}{\alpha_{Mn}^2 + \left(\frac{p\pi}{a}\right)^2} \left[(1 + \alpha_{Mn} a \coth \alpha_{Mn} a) \tanh \alpha_{Mn} a \right. \\
 &\quad \left. - \alpha_{Mn} a + \frac{\alpha_{Mn}^2 - \left(\frac{p\pi}{a}\right)^2}{\alpha_{Mn}^2 + \left(\frac{p\pi}{a}\right)^2} \tanh \alpha_{Mn} a \right]
 \end{aligned}$$

Put $l = p$ and simplify to give :

$$= \sum_M \frac{4H_{Mn} \alpha_{Mn}^3 a c \delta_1 (-1)^{l+(M-1)/2}}{\left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2\right]^2} \tanh \alpha_{Mn} a$$

where

$$\delta_1 = 2 \quad \text{when } n = 0, \quad \delta_1 = 1 \quad \text{when } n \neq 0$$

Taking term (II)

$$\begin{aligned}
 & \int_{-c}^{+c} \int_{-a}^{+a} (II) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\
 &= \sum_n \sum_l \frac{K_{nl} b}{\beta_{nl}^2 c^2 \sinh \beta_{nl} b} \left[2\nu n^2 \pi^2 \sinh \beta_{nl} b + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) \right. \right. \\
 &\quad \left. \left. \times \sinh \beta_{nl} b + \beta_{nl} b \cosh \beta_{nl} b \right\} \right] \int_{-a}^{+a} \cos \frac{l\pi x}{a} \cos \frac{p\pi x}{a} dx \int_{-c}^{+c} \cos \frac{n\pi z}{c} \cos \frac{q\pi z}{c} dz \\
 &= \frac{K_{nl} a b \delta_2}{\beta_{nl}^2 c \sinh \beta_{nl} b} \left[2\nu n^2 \pi^2 \sinh \beta_{nl} b + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) \sinh \beta_{nl} b + \beta_{nl} b \cosh \beta_{nl} b \right\} \right] \\
 &= \frac{K_{nl} a b \delta_2}{\beta_{nl}^2 c} \left[2\nu n^2 \pi^2 + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{nl} b \tanh \beta_{nl} b) + \beta_{nl} b \coth \beta_{nl} b \right\} \right]
 \end{aligned}$$

APPENDIX A

where

$$\delta_2 = 2 \quad \text{when } n = 0 \quad \text{or } l = 0, \quad \delta_2 = 1 \quad \text{when } n \neq 0, \quad l \neq 0$$

Taking term (III)

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{III}) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\ &= \sum_l \sum_M \frac{L_{lm} c (-1)^{(M-1)/2}}{\gamma_{lm}^2 b^2 \cosh \gamma_{lm} c} \left[\left\{ 2\nu \frac{M^2 \pi^2}{4} + \left(\frac{l\pi b}{a} \right)^2 (1 - \gamma_{lm} c \coth \gamma_{lm} c) \right\} \right. \\ & \quad \times \int_{-c}^{+c} \cosh \gamma_{lm} z \cos \frac{q\pi z}{c} dz + \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} \int_{-c}^{+c} z \sinh \gamma_{lm} z \cos \frac{q\pi z}{c} dz \Big] \\ & \quad \times \int_{-a}^{+a} \cos \frac{l\pi x}{a} \cos \frac{p\pi x}{a} dx \\ &= \sum_M \frac{L_{lm} \delta_3 a c (-1)^{(M-1)/2}}{\gamma_{lm}^2 b^2 \cosh \gamma_{lm} c} \left[\left\{ 2\nu \frac{M^2 \pi^2}{4} + \left(\frac{l\pi b}{a} \right)^2 (1 - \gamma_{lm} c \coth \gamma_{lm} c) \right\} \right. \\ & \quad \times \frac{2\gamma_{lm} (-1)^q}{\gamma_{lm}^2 + \left(\frac{q\pi}{c} \right)^2} \sinh \gamma_{lm} c + \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} \frac{2\gamma_{lm} c (-1)^q}{\gamma_{lm}^2 + \left(\frac{q\pi}{c} \right)^2} \cosh \gamma_{lm} c \\ & \quad \left. - \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} \frac{2(-1)^q \left[\gamma_{lm}^2 - \left(\frac{q\pi}{c} \right)^2 \right]}{\left[\gamma_{lm}^2 + \left(\frac{q\pi}{c} \right)^2 \right]^2} \sinh \gamma_{lm} c \right] \\ &= \sum_M \frac{2L_{lm} \delta_3 a c (-1)^{q+(M-1)/2}}{\gamma_{lm} b^2 \left[\gamma_{lm}^2 + \left(\frac{q\pi}{c} \right)^2 \right]} \left[\left\{ 2\nu \frac{M^2 \pi^2}{4} + \left(\frac{l\pi b}{a} \right)^2 (1 - \gamma_{lm} c \coth \gamma_{lm} c) \right\} \right. \\ & \quad \times \tanh \gamma_{lm} c + \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} c - \left(\frac{l\pi b}{a} \right)^2 \frac{\gamma_{lm}^2 - \left(\frac{q\pi}{c} \right)^2}{\gamma_{lm}^2 + \left(\frac{q\pi}{c} \right)^2} \tanh \gamma_{lm} c \Big] \end{aligned}$$

Put $n = q$, re-arrange and simplify to give:

$$= \sum_M \frac{4L_{lm} \delta_3 a c (-1)^{n+(M-1)/2}}{\gamma_{lm} b^2 \left[\gamma_{lm}^2 + \left(\frac{n\pi}{c} \right)^2 \right]} \left[\nu \frac{M^2 \pi^2}{4} + \left(\frac{l\pi b}{a} \right)^2 \frac{\left(\frac{n\pi}{c} \right)^2}{\gamma_{lm}^2 + \left(\frac{n\pi}{c} \right)^2} \right] \tanh \gamma_{lm} c$$

where

$$\delta_3 = 2 \quad \text{when } l = 0, \quad \delta_3 = 1 \quad \text{when } l \neq 0$$

APPENDIX A

The integrals of parts (IV) (V) and (VI) were derived using the same approach as shown in (I) (II) and (III).

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{IV}) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\ &= \sum_M \frac{4L_{1M}\delta_3\gamma_{1M}^3 ac(-1)^{n+(M-1)/2}}{\left[\gamma_{1M}^2 + \left(\frac{n\pi}{c}\right)^2\right]^2} \tanh \gamma_{1M} c \end{aligned}$$

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{V}) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\ &= \sum_M \frac{4H_{Mn}\delta_1 ac(-1)^{l+(M-1)/2}}{\alpha_{Mn} b^2 \left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2\right]} \left[\nu \frac{M^2 \pi^2}{4} + \left(\frac{n\pi b}{c}\right)^2 \frac{\left(\frac{l\pi}{a}\right)^2}{\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2} \right] \tanh \alpha_{Mn} a \end{aligned}$$

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{VI}) \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\ &= \frac{K_{n1} b c \delta_2}{\beta_{n1}^2 a} \left[2\nu l^2 \pi^2 + \left(\frac{n\pi a}{c}\right)^2 \left\{ (1 - \beta_{n1} b \tanh \beta_{n1} b) + \beta_{n1} b \coth \beta_{n1} b \right\} \right] \end{aligned}$$

where

$$\begin{aligned} \delta_1 &= 2 \quad \text{when } n = 0, \quad \delta_1 = 1 \quad \text{when } n \neq 0 \\ \delta_2 &= 2 \quad \text{when } n = 0 \text{ or } l = 0, \quad \delta_2 = 1 \quad \text{when } n \neq 0, \quad l \neq 0 \\ \delta_3 &= 2 \quad \text{when } l = 0, \quad \delta_3 = 1 \quad \text{when } l \neq 0 \end{aligned}$$

APPENDIX A

The individual integral components may now be summed to give the following.

$$\begin{aligned}
 & \int_{-c}^{+c} \int_{-a}^{+a} \left[(I) + (II) + (III) + (IV) + (V) + (VI) \right] \cos \frac{p\pi x}{a} \cos \frac{q\pi z}{c} dx dz \\
 &= \sum_M \frac{4H_{Mn}\delta_1\alpha_{Mn}^3ac(-1)^{l+(M-1)/2}}{\left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2\right]^2} \tanh \alpha_{Mn}a \\
 &+ \sum_M \frac{4H_{Mn}\delta_1ac(-1)^{l+(M-1)/2}}{\alpha_{Mn}b^2\left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2\right]} \left[\nu \frac{M^2\pi^2}{4} + \left(\frac{n\pi b}{c}\right)^2 \frac{\left(\frac{l\pi}{a}\right)^2}{\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2} \right] \tanh \alpha_{Mn}a \\
 &+ \frac{K_{n1}ab\delta_2}{\beta_{n1}^2c} \left[2\nu n^2\pi^2 + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{n1}b \tanh \beta_{n1}b) + \beta_{n1}b \coth \beta_{n1}b \right\} \right] \\
 &+ \frac{K_{n1}bc\delta_2}{\beta_{n1}^2a} \left[2\nu l^2\pi^2 + \left(\frac{n\pi a}{c}\right)^2 \left\{ (1 - \beta_{n1}b \tanh \beta_{n1}b) + \beta_{n1}b \coth \beta_{n1}b \right\} \right] \\
 &+ \sum_M \frac{4\delta_3L_{1M}ac(-1)^{n+(M-1)/2}}{\gamma_{1M}b^2\left[\gamma_{1M}^2 + \left(\frac{n\pi}{c}\right)^2\right]} \left[\nu \frac{M^2\pi^2}{4} + \left(\frac{l\pi b}{a}\right)^2 \frac{\left(\frac{n\pi}{c}\right)^2}{\gamma_{1M}^2 + \left(\frac{n\pi}{c}\right)^2} \right] \tanh \gamma_{1M}c \\
 &+ \sum_M \frac{4\delta_3L_{1M}\gamma_{1M}^3ac(-1)^{n+(M-1)/2}}{\left[\gamma_{1M}^2 + \left(\frac{n\pi}{c}\right)^2\right]^2} \tanh \gamma_{1M}c
 \end{aligned}$$

Simplifying this equation gives the following result, as presented in equation (22).

$$\begin{aligned}
 & \frac{1}{abc} \int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z) \Big|_{y=b} \cos \frac{l\pi x}{a} \cos \frac{n\pi z}{c} dx dz \\
 &= \sum_M \frac{4H_{Mn}\delta_1(-1)^{l+(M-1)/2}}{b\left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2\right]^2} \left[\alpha_{Mn}^3 + \frac{1}{\alpha_{Mn}b^2} \left\{ \nu \frac{M^2\pi^2}{4} \left[\alpha_{Mn}^2 + \left(\frac{l\pi}{a}\right)^2 \right] \right. \right. \\
 &\quad \left. \left. + \left(\frac{n\pi b}{c}\right)^2 \left(\frac{l\pi}{a}\right)^2 \right\} \right] \tanh \alpha_{Mn}a \\
 &+ \sum_M \frac{4\delta_3L_{1M}(-1)^{n+(M-1)/2}}{b\left[\gamma_{1M}^2 + \left(\frac{n\pi}{c}\right)^2\right]^2} \left[\gamma_{1M}^3 + \frac{1}{\gamma_{1M}b^2} \left\{ \nu \frac{M^2\pi^2}{4} \left[\gamma_{1M}^2 + \left(\frac{n\pi}{c}\right)^2 \right] \right. \right. \\
 &\quad \left. \left. + \left(\frac{l\pi b}{a}\right)^2 \left(\frac{n\pi}{c}\right)^2 \right\} \right] \tanh \gamma_{1M}c \\
 &+ \frac{K_{n1}\delta_2\pi^2}{\beta_{n1}^2a^2c^2} (a^2n^2 + c^2l^2) (2\nu + 1 - \beta_{n1}b \tanh \beta_{n1}b + \beta_{n1}b \coth \beta_{n1}b)
 \end{aligned}$$

DERIVATION OF EQUATION (24)

From Equation (23)

$$\begin{aligned}
\sigma_x + \sigma_z = & \sum_m \sum_N \frac{H_m N a}{\cosh \alpha_m N a} \left\{ (1 + \alpha_m N a \coth \alpha_m N a) \cosh \alpha_m N x - \alpha_m N x \sinh \alpha_m N x \right\} \\
& \times \cos \frac{m \pi y}{b} \sin \frac{N \pi z}{2c} + \sum_N \sum_l \frac{K_{Nl} b}{\beta_{Nl}^2 c^2 \cosh \beta_{Nl} b} \left[2\nu \frac{N^2 \pi^2}{4} \cosh \beta_{Nl} y + \left(\frac{l \pi c}{a} \right)^2 \right. \\
& \times \left. \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) \cosh \beta_{Nl} y + \beta_{Nl} y \sinh \beta_{Nl} y \right\} \right] \sin \frac{N \pi z}{2c} \cos \frac{l \pi x}{a} \\
& + \sum_l \sum_m \frac{L_{lm} c}{\gamma_{lm}^2 b^2 \sinh \gamma_{lm} c} \left[2\nu m^2 \pi^2 \sinh \gamma_{lm} z + \left(\frac{l \pi b}{a} \right)^2 \left\{ (1 - \gamma_{lm} c \tanh \gamma_{lm} c) \right. \right. \\
& \times \left. \left. \sinh \gamma_{lm} z + \gamma_{lm} z \cosh \gamma_{lm} z \right\} \right] \cos \frac{l \pi x}{a} \cos \frac{m \pi y}{b} \\
& + \sum_l \sum_m \frac{L_{lm} c}{\sinh \gamma_{lm} c} \left\{ (1 + \gamma_{lm} c \tanh \gamma_{lm} c) \sinh \gamma_{lm} z - \gamma_{lm} z \cosh \gamma_{lm} z \right\} \\
& \times \cos \frac{l \pi x}{a} \cos \frac{m \pi y}{b} + \sum_m \sum_N \frac{H_m N a}{\alpha_m^2 N b^2 \cosh \alpha_m N a} \left[2\nu m^2 \pi^2 \cosh \alpha_m N x + \left(\frac{N \pi b}{2c} \right)^2 \right. \\
& \times \left. \left\{ (1 - \alpha_m N a \coth \alpha_m N a) \cosh \alpha_m N x + \alpha_m N x \sinh \alpha_m N x \right\} \right] \cos \frac{m \pi y}{b} \sin \frac{N \pi z}{2c} \\
& + \sum_N \sum_l \frac{K_{Nl} b}{\beta_{Nl}^2 a^2 \cosh \beta_{Nl} b} \left[2\nu l^2 \pi^2 \cosh \beta_{Nl} y + \left(\frac{N \pi a}{2c} \right)^2 \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) \right. \right. \\
& \times \left. \left. \cosh \beta_{Nl} y + \beta_{Nl} y \sinh \beta_{Nl} y \right\} \right] \sin \frac{N \pi z}{2c} \cos \frac{l \pi x}{a}
\end{aligned}$$

On a lateral surface of $y = b$, $\sigma_y = 0$ and so the bulk stress on this surface becomes $\sigma_b = \sigma_x + \sigma_z$. Putting $y = b$ into the above equation, and noting that $\cos m\pi = (-1)^m$ for $m = 0, 1, 2, \dots$, yields the following result.

APPENDIX B

$$\sigma_b = \sum_m \sum_N \frac{H_{mNa}(-1)^m}{\cosh \alpha_{mNa}} \left\{ (1 + \alpha_{mNa} \coth \alpha_{mNa}) \cosh \alpha_{mNx} - \alpha_{mNx} \sinh \alpha_{mNx} \right\} \sin \frac{N\pi z}{2c} \quad (I)$$

$$+ \sum_N \sum_l \frac{K_{Nl}b}{\beta_{Nl}^2 c^2 \cosh \beta_{Nl}b} \left[2\nu \frac{N^2 \pi^2}{4} \cosh \beta_{Nl}b + \left(\frac{l\pi c}{a} \right)^2 \left\{ (1 - \beta_{Nl}b \coth \beta_{Nl}b) \right. \right. \\ \left. \left. \times \cosh \beta_{Nl}b + \beta_{Nl}b \sinh \beta_{Nl}b \right\} \right] \sin \frac{N\pi z}{2c} \cos \frac{l\pi x}{a} \quad (II)$$

$$+ \sum_l \sum_m \frac{L_{lm}c(-1)^m}{\gamma_{lm}^2 b^2 \sinh \gamma_{lm}c} \left[2\nu m^2 \pi^2 \sinh \gamma_{lm}z + \left(\frac{l\pi b}{a} \right)^2 \left\{ (1 - \gamma_{lm}c \tanh \gamma_{lm}c) \right. \right. \\ \left. \left. \times \sinh \gamma_{lm}z + \gamma_{lm}z \cosh \gamma_{lm}z \right\} \right] \cos \frac{l\pi x}{a} \quad (III)$$

$$+ \sum_l \sum_m \frac{L_{lm}c(-1)^m}{\sinh \gamma_{lm}c} \left\{ (1 + \gamma_{lm}c \tanh \gamma_{lm}c) \sinh \gamma_{lm}z - \gamma_{lm}z \cosh \gamma_{lm}z \right\} \cos \frac{l\pi x}{a} \quad (IV)$$

$$+ \sum_m \sum_N \frac{H_{mNa}(-1)^m}{\alpha_{mN}^2 b^2 \cosh \alpha_{mNa}} \left[2\nu m^2 \pi^2 \cosh \alpha_{mNx} + \left(\frac{N\pi b}{2c} \right)^2 \left\{ (1 - \alpha_{mNa} \coth \alpha_{mNa}) \right. \right. \\ \left. \left. \times \cosh \alpha_{mNx} + \alpha_{mNx} \sinh \alpha_{mNx} \right\} \right] \sin \frac{N\pi z}{2c} \quad (V)$$

$$+ \sum_N \sum_l \frac{K_{Nl}b}{\beta_{Nl}^2 a^2 \cosh \beta_{Nl}b} \left[2\nu l^2 \pi^2 \cosh \beta_{Nl}b + \left(\frac{N\pi a}{2c} \right)^2 \left\{ (1 - \beta_{Nl}b \coth \beta_{Nl}b) \right. \right. \\ \left. \left. \times \cosh \beta_{Nl}b + \beta_{Nl}b \sinh \beta_{Nl}b \right\} \right] \sin \frac{N\pi z}{2c} \cos \frac{l\pi x}{a} \quad (VI)$$

To determine the Fourier coefficients H_{mN} , K_{Nl} and L_{lm} it is necessary to take the double Fourier transform of the above equation.

$$\int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\ = \int_{-c}^{+c} \int_{-a}^{+a} \left[(I) + (II) + (III) + (IV) + (V) + (VI) \right] \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz$$

Note that p and Q are now fixed positive integers.

APPENDIX B

Integrating the RHS, taking term (I)

$$\begin{aligned}
 & \int_{-c}^{+c} \int_{-a}^{+a} (I) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\
 &= \sum_m \sum_N \frac{H_{mNa}(-1)^m}{\cosh \alpha_{mNa}} \left[(1 + \alpha_{mNa} \coth \alpha_{mNa}) \int_{-a}^{+a} \cosh \alpha_{mNx} \cos \frac{p\pi x}{a} dx \right. \\
 &\quad \left. - \alpha_{mN} \int_{-a}^{+a} x \sinh \alpha_{mNx} \cos \frac{p\pi x}{a} dx \right] \int_{-c}^{+c} \sin \frac{N\pi z}{2c} \sin \frac{Q\pi z}{2c} dz \\
 &= \sum_m \frac{H_{mNa}(-1)^m}{\cosh \alpha_{mNa}} \left[(1 + \alpha_{mNa} \coth \alpha_{mNa}) \frac{2\alpha_{mN}(-1)^p}{\alpha_{mN}^2 + \left(\frac{p\pi}{a}\right)^2} \sinh \alpha_{mNa} \right. \\
 &\quad \left. - \frac{2\alpha_{mNa}^2(-1)^p}{\alpha_{mN}^2 + \left(\frac{p\pi}{a}\right)^2} \cosh \alpha_{mNa} + \frac{2\alpha_{mN}(-1)^p \left[\alpha_{mN}^2 - \left(\frac{p\pi}{a}\right)^2 \right]}{\left[\alpha_{mN}^2 + \left(\frac{p\pi}{a}\right)^2 \right]^2} \sinh \alpha_{mNa} \right] \\
 &= \sum_m \frac{2H_{mNa} \alpha_{mNa}(-1)^{m+p}}{\alpha_{mN}^2 + \left(\frac{p\pi}{a}\right)^2} \left[(1 + \alpha_{mNa} \coth \alpha_{mNa}) \tanh \alpha_{mNa} \right. \\
 &\quad \left. - \alpha_{mNa} + \frac{\alpha_{mN}^2 - \left(\frac{p\pi}{a}\right)^2}{\alpha_{mN}^2 + \left(\frac{p\pi}{a}\right)^2} \tanh \alpha_{mNa} \right]
 \end{aligned}$$

Put $l = p$ and simplify to give :

$$= \sum_m \frac{4H_{mNa} \alpha_{mNa}^3(-1)^{m+l}}{\left[\alpha_{mN}^2 + \left(\frac{l\pi}{a}\right)^2 \right]^2} \tanh \alpha_{mNa}$$

Taking term (II)

$$\begin{aligned}
 & \int_{-c}^{+c} \int_{-a}^{+a} (II) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\
 &= \sum_N \sum_l \frac{K_{Nb}}{\beta_{Nl}^2 c^2 \cosh \beta_{Nl} b} \left[2\nu \frac{N^2 \pi^2}{4} \cosh \beta_{Nl} b + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) \right. \right. \\
 &\quad \left. \left. \times \cosh \beta_{Nl} b + \beta_{Nl} b \sinh \beta_{Nl} b \right\} \right] \int_{-a}^{+a} \cos \frac{l\pi x}{a} \cos \frac{p\pi x}{a} dx \int_{-c}^{+c} \sin \frac{N\pi z}{2c} \sin \frac{Q\pi z}{2c} dz \\
 &= \frac{K_{Nab} \delta_1}{\beta_{Nl}^2 c \cosh \beta_{Nl} b} \left[2\nu \frac{N^2 \pi^2}{4} \cosh \beta_{Nl} b + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) \cosh \beta_{Nl} b + \beta_{Nl} b \sinh \beta_{Nl} b \right\} \right] \\
 &= \frac{K_{Nab} \delta_1}{\beta_{Nl}^2 c} \left[2\nu \frac{N^2 \pi^2}{4} + \left(\frac{l\pi c}{a}\right)^2 \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) + \beta_{Nl} b \tanh \beta_{Nl} b \right\} \right]
 \end{aligned}$$

APPENDIX B

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

Taking term (III)

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{III}) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\ &= \sum_l \sum_m \frac{L_{lm} c (-1)^m}{\gamma_{lm}^2 b^2 \sinh \gamma_{lm} c} \left[\left\{ 2\nu m^2 \pi^2 + \left(\frac{l\pi b}{a} \right)^2 (1 - \gamma_{lm} c \tanh \gamma_{lm} c) \right\} \right. \\ & \quad \times \int_{-c}^{+c} \sinh \gamma_{lm} z \sin \frac{Q\pi z}{2c} dz + \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} \int_{-c}^{+c} z \cosh \gamma_{lm} z \sin \frac{Q\pi z}{2c} dz \Big] \\ & \quad \times \int_{-a}^{+a} \cos \frac{l\pi x}{a} \cos \frac{p\pi x}{a} dx \\ &= \sum_m \frac{L_{lm} \delta_1 a c (-1)^m}{\gamma_{lm}^2 b^2 \sinh \gamma_{lm} c} \left[\left\{ 2\nu m^2 \pi^2 + \left(\frac{l\pi b}{a} \right)^2 (1 - \gamma_{lm} c \tanh \gamma_{lm} c) \right\} \right. \\ & \quad \times \frac{2\gamma_{lm} (-1)^{(Q-1)/2}}{\gamma_{lm}^2 + \left(\frac{Q\pi}{2c} \right)^2} \cosh \gamma_{lm} c + \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} \frac{2\gamma_{lm} c (-1)^{(Q-1)/2}}{\gamma_{lm}^2 + \left(\frac{Q\pi}{2c} \right)^2} \sinh \gamma_{lm} c \\ & \quad \left. - \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} \frac{2(-1)^{(Q-1)/2} \left[\gamma_{lm}^2 - \left(\frac{Q\pi}{2c} \right)^2 \right]}{\left[\gamma_{lm}^2 + \left(\frac{Q\pi}{2c} \right)^2 \right]^2} \cosh \gamma_{lm} c \right] \\ &= \sum_m \frac{2L_{lm} \delta_1 a c (-1)^{m+(Q-1)/2}}{\gamma_{lm} b^2 \left[\gamma_{lm}^2 + \left(\frac{Q\pi}{2c} \right)^2 \right]} \left[\left\{ 2\nu m^2 \pi^2 + \left(\frac{l\pi b}{a} \right)^2 (1 - \gamma_{lm} c \tanh \gamma_{lm} c) \right\} \right. \\ & \quad \times \coth \gamma_{lm} c + \left(\frac{l\pi b}{a} \right)^2 \gamma_{lm} c - \left(\frac{l\pi b}{a} \right)^2 \frac{\gamma_{lm}^2 - \left(\frac{Q\pi}{2c} \right)^2}{\gamma_{lm}^2 + \left(\frac{Q\pi}{2c} \right)^2} \coth \gamma_{lm} c \Big] \end{aligned}$$

Put $N = Q$, re-arrange and simplify to give:

$$= \sum_m \frac{4\delta_1 L_{lm} a c (-1)^{m+(N-1)/2}}{\gamma_{lm} b^2 \left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right]} \left[\nu m^2 \pi^2 + \left(\frac{l\pi b}{a} \right)^2 \frac{\left(\frac{N\pi}{2c} \right)^2}{\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2} \right] \coth \gamma_{lm} c$$

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

APPENDIX B

The integrals of parts (IV) (V) and (VI) were derived using the same approach as shown in (I) (II) and (III).

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{IV}) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\ &= \sum_m \frac{4\delta_1 L_{lm} \gamma_{lm}^3 ac (-1)^{m+(N-1)/2}}{\left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right]^2} \coth \gamma_{lm} c \end{aligned}$$

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{V}) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\ &= \sum_m \frac{4H_{mN} ac (-1)^{m+l}}{\alpha_{mN} b^2 \left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right]} \left[\nu m^2 \pi^2 + \left(\frac{N\pi b}{2c} \right)^2 \frac{\left(\frac{l\pi}{a} \right)^2}{\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2} \right] \tanh \alpha_{mN} a \end{aligned}$$

$$\begin{aligned} & \int_{-c}^{+c} \int_{-a}^{+a} (\text{VI}) \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\ &= \frac{K_{Nl} bc \delta_1}{\beta_{Nl}^2 a} \left[2\nu l^2 \pi^2 + \left(\frac{N\pi a}{2c} \right)^2 \left\{ (1 - \beta_{Nl} b \coth \beta_{Nl} b) + \beta_{Nl} b \tanh \beta_{Nl} b \right\} \right] \end{aligned}$$

where

$$\delta_1 = 2 \quad \text{when } l = 0, \quad \delta_1 = 1 \quad \text{when } l \neq 0$$

APPENDIX B

The individual integral components may now be summed to give the following.

$$\begin{aligned}
 & \int_{-c}^{+c} \int_{-a}^{+a} \left[(I) + (II) + (III) + (IV) + (V) + (VI) \right] \cos \frac{p\pi x}{a} \sin \frac{Q\pi z}{2c} dx dz \\
 &= \sum_m \frac{4H_m N \alpha_m^3 \alpha c (-1)^{m+l}}{\left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right]^2} \tanh \alpha_{mN} a \\
 &+ \sum_m \frac{4H_m N \alpha c (-1)^{m+l}}{\alpha_{mN} b^2 \left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right]} \left[\nu m^2 \pi^2 + \left(\frac{N\pi b}{2c} \right)^2 \frac{\left(\frac{l\pi}{a} \right)^2}{\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2} \right] \tanh \alpha_{mN} a \\
 &+ \frac{K_{NI} a b \delta_1}{\beta_{NI}^2 c} \left[2\nu \frac{N^2 \pi^2}{4} + \left(\frac{l\pi c}{a} \right)^2 \left\{ (1 - \beta_{NI} b \coth \beta_{NI} b) + \beta_{NI} b \tanh \beta_{NI} b \right\} \right] \\
 &+ \frac{K_{NI} b c \delta_1}{\beta_{NI}^2 a} \left[2\nu l^2 \pi^2 + \left(\frac{N\pi a}{2c} \right)^2 \left\{ (1 - \beta_{NI} b \coth \beta_{NI} b) + \beta_{NI} b \tanh \beta_{NI} b \right\} \right] \\
 &+ \sum_m \frac{4\delta_1 L_{lm} \alpha c (-1)^{m+(N-1)/2}}{\gamma_{lm} b^2 \left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right]} \left[\nu m^2 \pi^2 + \left(\frac{l\pi b}{a} \right)^2 \frac{\left(\frac{N\pi}{2c} \right)^2}{\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2} \right] \coth \gamma_{lm} c \\
 &+ \sum_m \frac{4\delta_1 L_{lm} \gamma_{lm}^3 \alpha c (-1)^{m+(N-1)/2}}{\left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right]^2} \coth \gamma_{lm} c
 \end{aligned}$$

Simplifying this equation gives the following result, as presented in equation (24).

$$\begin{aligned}
 & \frac{1}{abc} \int_{-c}^{+c} \int_{-a}^{+a} (\sigma_x + \sigma_z) \Big|_{y=b} \cos \frac{l\pi x}{a} \sin \frac{N\pi z}{2c} dx dz \\
 &= \sum_m \frac{4H_m N (-1)^{m+l}}{b \left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right]^2} \left[\alpha_{mN}^3 + \frac{1}{\alpha_{mN} b^2} \left\{ \nu m^2 \pi^2 \left[\alpha_{mN}^2 + \left(\frac{l\pi}{a} \right)^2 \right] \right. \right. \right. \\
 &\quad \left. \left. \left. + \left(\frac{N\pi b}{2c} \right)^2 \left(\frac{l\pi}{a} \right)^2 \right\} \right] \tanh \alpha_{mN} a \\
 &+ \sum_m \frac{4\delta_1 L_{lm} (-1)^{m+(N-1)/2}}{b \left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right]^2} \left[\gamma_{lm}^3 + \frac{1}{\gamma_{lm} b^2} \left\{ \nu m^2 \pi^2 \left[\gamma_{lm}^2 + \left(\frac{N\pi}{2c} \right)^2 \right] \right. \right. \right. \\
 &\quad \left. \left. \left. + \left(\frac{l\pi b}{a} \right)^2 \left(\frac{N\pi}{2c} \right)^2 \right\} \right] \coth \gamma_{lm} c \\
 &+ \frac{K_{NI} \delta_1 \pi^2}{\beta_{NI}^2 a^2 c^2} \left[\frac{N^2 a^2}{4} + c^2 l^2 \right] \left(2\nu + 1 - \beta_{NI} b \coth \beta_{NI} b + \beta_{NI} b \tanh \beta_{NI} b \right)
 \end{aligned}$$

DISTRIBUTION

AUSTRALIA

Department of Defence

Defence Central

Chief Defence Scientist)
AS, Science Corporate Management)shared copy
FAS Science Policy)
Director, Departmental Publications
Counsellor, Defence Science, London (Doc Data sheet only)
Counsellor, Defence Science, Washington (Doc Data sheet only)
Scientific Adviser, Defence Central
OIC TRS, Defence Central Library
Document Exchange Centre, DSTIC (8 copies)
Defence Intelligence Organisation
Librarian H Block, Victoria Barracks, Melb (Doc Data sheet only)

Aeronautical Research Laboratory

Director
Library
Chief - Aircraft Structures Division
Divisional File - Aircraft Structures
Author: J. Bennett
R. Callinan
W. Chiu
M. Heller
R. Jones
R. Kaye
L. Molent
J. Paul
D. Rees
T. Tran-Cong

Navy Scientific Adviser (3 copies Doc Data sheet only)

Scientific Adviser - Army (Doc Data sheet only)

Air Force Scientific Adviser

SPARES (10 COPIES)

TOTAL (38 COPIES)

DOCUMENT CONTROL DATAPAGE CLASSIFICATION
UNCLASSIFIED

PRIVACY MARKING

1a. AR NUMBER AR-006-621	1b. ESTABLISHMENT NUMBER ARL-STRUC-TM-539	2. DOCUMENT DATE SEPTEMBER 1991	3. TASK NUMBER AIR 91/056
4. TITLE DETERMINATION OF ANTISYMMETRIC END LOADS FROM BULK STRESSES ON A RECTANGULAR PRISM		5. SECURITY CLASSIFICATION (PLACE APPROPRIATE CLASSIFICATION IN BOXES (E. SECRET (S), CONF. (C) RESTRICTED (R), UNCLASSIFIED (U)). <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 2px; text-align: center;">U</div> <div style="border: 1px solid black; padding: 2px; text-align: center;">U</div> <div style="border: 1px solid black; padding: 2px; text-align: center;">U</div> </div> DOCUMENT TITLE ABSTRACT	6. NO. PAGES 39
		7. NO. REFS. 2	
8. AUTHOR(S) J. BENNETT		9. DOWNGRADING/DELIMITING INSTRUCTIONS Not applicable	
10. CORPORATE AUTHOR AND ADDRESS AERONAUTICAL RESEARCH LABORATORY 506 LORIMER STREET FISHERMENS BEND VIC 3207		11. OFFICE/POSITION RESPONSIBLE FOR: SPONSOR RAAF SECURITY - DOWNGRADING - APPROVAL CSTD	
12. SECONDARY DISTRIBUTION (OF THIS DOCUMENT) Approved for public release OVERSEAS ENQUIRIES OUTSIDE STATED LIMITATIONS SHOULD BE REFERRED THROUGH DSTIC, ADMINISTRATIVE SERVICES BRANCH, DEPARTMENT OF DEFENCE, ANZAC PARK WEST OFFICES, ACT 2601			
13a. THIS DOCUMENT MAY BE ANNOUNCED IN CATALOGUES AND AWARENESS SERVICES AVAILABLE TO No limitations			
13b. CITATION FOR OTHER PURPOSES (E. CASUAL ANNOUNCEMENT) MAY BE <input checked="" type="checkbox"/> UNRESTRICTED OR <input type="checkbox"/> AS FOR 13a.			
14. DESCRIPTORS Loading Stress analysis		15. DISCAT SUBJECT CATEGORIES 2011	
16. ABSTRACT <i>A mathematical method is presented for the determination of the normal antisymmetric end loads from known values of the bulk stress on a lateral surface of a three dimensional rectangular prism. This extends previous work which considered only the symmetric loading case. A computer program has been developed to implement this method, and results for an arbitrarily chosen case are presented.</i>			

PAGE CLASSIFICATION
UNCLASSIFIED

PRIVACY MARKING

THIS PAGE IS TO BE USED TO RECORD INFORMATION WHICH IS REQUIRED BY THE ESTABLISHMENT FOR ITS OWN USE BUT WHICH WILL NOT BE ADDED TO THE DISTIS DATA UNLESS SPECIFICALLY REQUESTED.

16. ABSTRACT (CONT).

17. IMPRINT

AERONAUTICAL RESEARCH LABORATORY, MELBOURNE

18. DOCUMENT SERIES AND NUMBER

Aircraft Structures Technical
Memorandum 539

19. COST CODE

21 218F

20. TYPE OF REPORT AND PERIOD COVERED

21. COMPUTER PROGRAMS USED

22. ESTABLISHMENT FILE REF.(S)

23. ADDITIONAL INFORMATION (AS REQUIRED)